

The Effective Theory of Inflation and the Dark Matter Status in the Standard Model of the Universe

H. J. de Vega ^{(a,b)*}

^(a) *LPTHE, Laboratoire Associé au CNRS UMR 7589,
Université Pierre et Marie Curie (Paris VI) et Denis Diderot (Paris VII),
Tour 24, 5^{ème}. étage, 4, Place Jussieu, 75252 Paris, Cedex 05, France.*

^(b) *Observatoire de Paris, LERMA, Laboratoire Associé au CNRS UMR 8112,
61, Avenue de l'Observatoire, 75014 Paris, France*

(Dated: June 4, 2010)

Inflation is today a part of the Standard Model of the Universe supported by the cosmic microwave background (CMB) and large scale structure (LSS) datasets. Inflation solves the horizon and flatness problems and naturally generates density fluctuations that seed LSS and CMB anisotropies, and tensor perturbations (primordial gravitational waves). Inflation theory is based on a scalar field φ (the inflaton) whose potential is fairly flat leading to a slow-roll evolution. We present here the effective theory of inflation à la Ginsburg-Landau in which the inflaton potential is a polynomial in the field φ and has the universal form $V(\varphi) = N M^4 w(\varphi/[\sqrt{N} M_{Pl}])$, where $w = \mathcal{O}(1)$, $M \ll M_{Pl}$ is the scale of inflation and $N \sim 60$ is the number of e-folds since the cosmologically relevant modes exit the horizon till inflation ends. The slow-roll expansion becomes a systematic $1/N$ expansion and the inflaton couplings become **naturally small** as powers of the ratio $(M/M_{Pl})^2$. The spectral index and the ratio of tensor/scalar fluctuations are $n_s - 1 = \mathcal{O}(1/N)$, $r = \mathcal{O}(1/N)$ while the running index turns to be $dn_s/d\ln k = \mathcal{O}(1/N^2)$ and therefore can be neglected. The energy scale of inflation $M \sim 0.7 \times 10^{16}$ GeV is completely determined by the amplitude of the scalar adiabatic fluctuations. A complete analytic study plus the Monte Carlo Markov Chains (MCMC) analysis of the available CMB+LSS data (including WMAP5) with fourth degree potentials showed: (a) the **spontaneous breaking** of the $\varphi \rightarrow -\varphi$ symmetry of the inflaton potential. (b) a **lower bound** for r in new inflation: $r > 0.023$ (95% CL) and $r > 0.046$ (68% CL). (c) The preferred inflation potential is a **double well**, even function of the field with a moderate quartic coupling yielding as most probable values: $n_s \simeq 0.964$, $r \simeq 0.051$. This value for r is within reach of forthcoming CMB observations. Study of higher degree inflaton potentials show that terms of degree higher than four do not affect the fit in a significant way. The initial conditions for the quantum fluctuations must be vacuum type (Bunch-Davies) in order to reproduce the CMB and LSS data. Slow-roll inflation is generically preceded by a short **fast-roll** stage. If the modes which are horizon-size today exited the horizon during fast-roll or at the transition between fast and slow-roll, the curvature and tensor CMB quadrupoles get suppressed in agreement with the CMB data for the former. Fast-roll fits the TT, the TE and the EE modes well reproducing the quadrupole suppression. A thorough study of the quantum loop corrections reveals that they are very small and controlled by powers of $(H/M_{Pl})^2 \sim 10^{-9}$, a conclusion that validates the reliability of the effective theory of inflation. Our work shows how powerful is the Ginsburg-Landau effective theory of inflation in predicting observables that are being or will soon be contrasted to observations. Dark matter (DM) constitutes 83 % of the matter in the Universe. We investigate the DM properties using cosmological theory and the galaxy observations from DM-dominated galaxies. Our DM analysis is independent of the particle physics model for DM and it is based on the DM phase-space density ρ_{DM}/σ_{DM}^3 . We derive explicit formulas for the DM particle mass m and for the number of ultrarelativistic degrees of freedom g_d (hence the temperature) at decoupling. We find that m turns to be at the **keV scale**. The keV scale DM is non-relativistic during structure formation, reproduces the small and large scale structure but it cannot be responsible of the e^+ and \bar{p} excess in cosmic rays which can be explained by astrophysical mechanisms.

Contents

I. Overview and present status of the Effective Theory of Inflation	2
II. The Standard Model of the Universe	7

*Electronic address: devega@lpthe.jussieu.fr

III. Inflation and Inflaton field dynamics	10
A. Slow-roll, the Universal Form of the Inflaton Potential and the Energy Scale of Inflation	14
B. The energy scale of inflation and the quasi-scale invariance during inflation.	16
C. Ginsburg-Landau polynomial realizations of the Inflaton Potential	19
IV. Overview of Dark Matter	19
A. The dark matter particle mass	20
B. Jeans' (free-streaming) wavelength and Jeans' mass	22
C. Dark Matter Decoupling at Local Thermal Equilibrium (LTE)	23
D. The DM particle at the keV scale: conclusions	24
V. Outlook and future perspectives	25
References	26

I. OVERVIEW AND PRESENT STATUS OF THE EFFECTIVE THEORY OF INFLATION

This article is dedicated to my colleague and friend Lev Lipatov in the occasion of his 70th birthdate. The content of this paper is not directly related to the monumental work of Lev in particle physics and field theory. However, the deep subtleties in the field theoretical treatments in cosmology deserve to include this contribution in Lev's Festschrift.

Inflation was introduced to solve several outstanding problems of the standard Big Bang model [1] and has now become an essential part of the standard cosmology. It provides a natural mechanism for the generation of scalar density fluctuations that seed large scale structure, thus explaining the origin of the temperature anisotropies in the cosmic microwave background (CMB), and for the generation of tensor perturbations (primordial gravitational waves) [2–5].

A distinct aspect of inflationary perturbations is that they are generated by quantum fluctuations of the scalar field(s) that drive inflation. After their wavelength becomes larger than the Hubble radius, these fluctuations are amplified and grow, becoming classical and decoupling from causal microphysical processes. Upon re-entering the horizon, during the radiation and matter dominated eras, these classical perturbations seed the inhomogeneities which generate structure upon gravitational collapse [3, 4]. These fluctuations enjoy fairly generic features: a gaussian, nearly scale invariant spectrum of adiabatic scalar and tensor primordial fluctuations, which provide an excellent fit to the highly precise wealth of CMB [8] and LSS data, making the inflationary paradigm robust. Precision CMB data reveal peaks and valleys in the temperature fluctuations resulting from acoustic oscillations in the electron-photon fluid at recombination. These are depicted in fig. 1 where up to five peaks can be seen.

Baryon acoustic oscillations driven by primordial fluctuations produce a peak in the galaxy correlations at $\sim 109 h^{-1}$ Mpc (comoving sound horizon) [6]. This peak is the real-space version of the acoustic oscillations in momentum (or l) space and are confirmed by LSS data [6].

Perhaps the most striking validation of inflation as a mechanism for generating *superhorizon* fluctuations is the anticorrelation peak in the temperature-polarization (TE) angular power spectrum at $l \sim 150$ corresponding to superhorizon scales [7] and depicted in fig. 2. The observed TE power spectrum can only be generated by fluctuations that exited the horizon during inflation and re-entered the horizon later, when the expansion of the universe decelerates.

The confirmation of many of the robust predictions of inflation by current high precision observations places inflationary cosmology on solid grounds.

Amongst the wide variety of inflationary scenarios, single field slow-roll models provide an appealing, simple and fairly generic description of inflation. Its implementation is based on a scalar field (the inflaton) whose homogeneous expectation value drives the dynamics of the scale factor, plus small quantum fluctuations. The inflaton potential is fairly flat during inflation and it dominates the universe energy during inflation. This flatness not only leads to a slowly varying Hubble parameter, hence ensuring a sufficient number of efolds of inflation, but also provides an explanation for the gaussianity of the fluctuations as well as for the (almost) scale invariance of their power spectrum. A flat potential precludes large non-linearities in the dynamics of the *fluctuations* of the scalar field.

The current WMAP data are validating the single field slow-roll scenario [7, 8]. Furthermore, because the potential is flat the scalar field is almost **massless**, and modes cross the horizon with an amplitude proportional to the Hubble parameter. This fact combined with a slowly varying Hubble parameter yields an almost scale invariant primordial

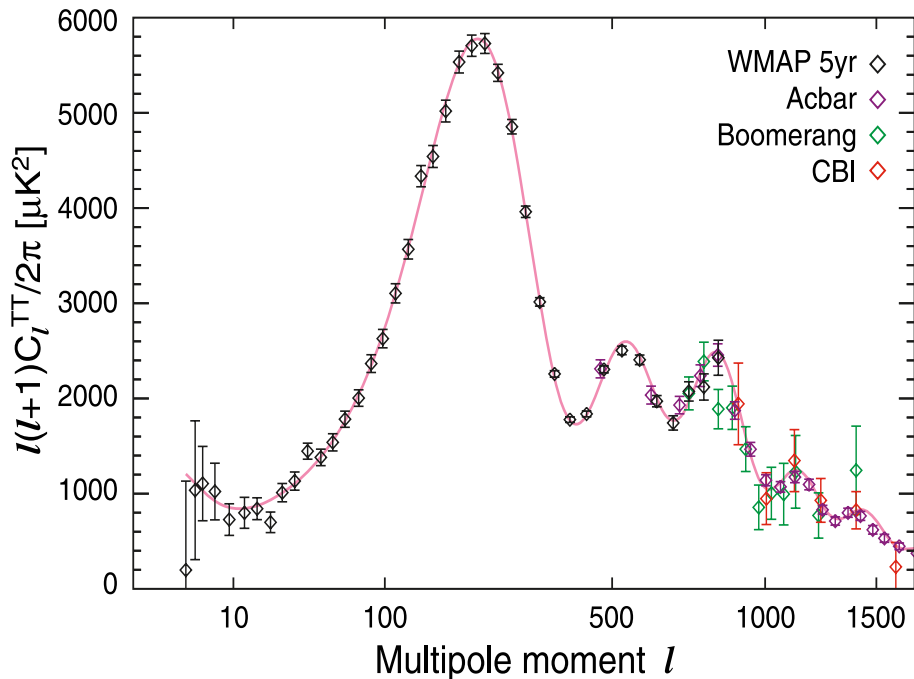


FIG. 1: Acoustic oscillations from WMAP 5 years data set plus other CMB data. Theory and observations nicely agree except for the lowest multipoles: the quadrupole CMB suppression. See refs. [19–21] and [5] for discussions on this.

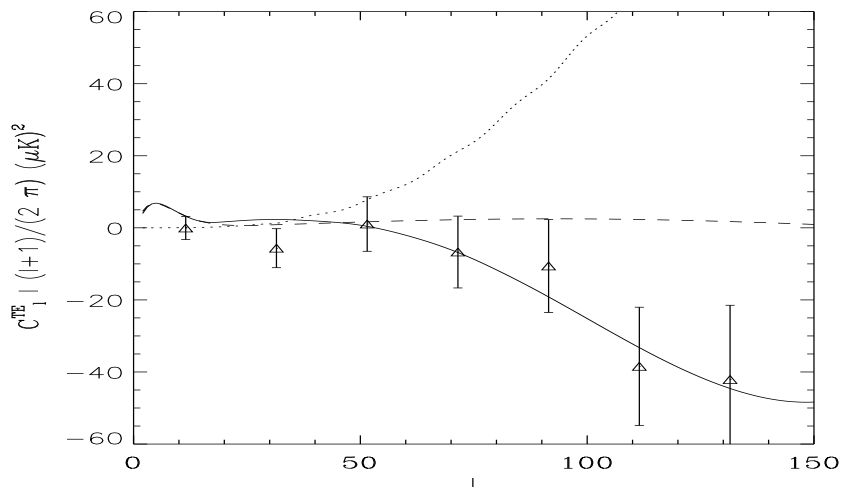


FIG. 2: Temperature-Polarization angular power spectrum. The large-angle TE power spectrum predicted in primordial adiabatic models (solid), primordial isocurvature models (dashed) and by defects such as cosmic strings (dotted). The WMAP TE data (Kogut et al. [7]) are shown for comparison, in bins of $\Delta l = 10$. Superhorizon adiabatic modes from inflation fit the data while subhorizon sources of TE power go in directions opposite to the data. Hence, we concentrate here and in ref. [5] on superhorizon adiabatic modes.

power spectrum. The slow-roll approximation has been cast as a systematic $1/N$ expansion [9], where $N \sim 60$ is the number of e-folds since the cosmologically relevant modes exited the horizon till the end of inflation.

The observational progress discriminates among different inflationary models, placing stringent constraints on them. The upper bound on the ratio r of tensor to scalar fluctuations obtained by WMAP convincingly excludes the massless monomial φ^4 potential [8] and hence **strongly suggests** the presence of a **mass term** in the single field inflaton potential [5, 10, 11]. Therefore, as a minimal single field model, one should consider a sufficiently general polynomial, the simplest polynomial potential bounded from below being the fourth degree potential [5, 10, 12].

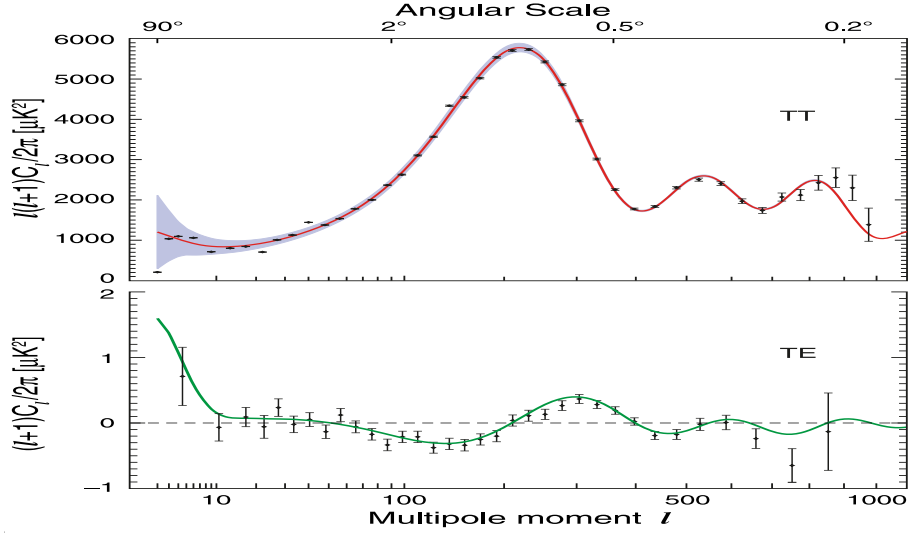


FIG. 3: The temperature (TT) and temperature-polarization correlation (TE) power spectra based on the 5 year WMAP data [8].

The observed low value of the CMB quadrupole with respect to the Λ CDM theoretical value has been an intriguing feature on large angular scales since first observed by COBE/DMR [13], and confirmed by the WMAP data [8]. In the best fit Λ CDM model, using the WMAP5 data we find that the probability that the quadrupole is as low or lower than the observed value is just 0.031 [5, 45]. Even if one does not care about the specific multipole and looks for any multipole as low or lower than the observed quadrupole with respect to the Λ CDM model value, then the probability remains smaller than 5%. Therefore, it is relevant to find a cosmological explanation of the quadrupole suppression beyond the Λ CDM model. An early fast-roll stage can explain the CMB quadrupole suppression as we discuss below.

The main new aspects of inflationary cosmology can be summarized as follows [5]:

- An effective field theory description of slow-roll single field inflation à la Ginsburg-Landau. In the Ginsburg-Landau framework, the potential is a polynomial in the field starting by a constant term [14]. Linear terms can always be eliminated by a constant shift of the inflaton field. The quadratic term can have a positive or a negative sign associated to chaotic or new inflation, respectively. This effective Ginsburg-Landau field theory is characterized by only **two** energy scales: the scale of inflation M and the Planck scale $M_{Pl} = 2.43534 \cdot 10^{18}$ GeV $\gg M$. In this context we propose a **universal** form for the inflaton potential in slow-roll models [9]:

$$V(\varphi) = N M^4 w(\chi) , \quad (1.1)$$

where $N \sim 60$ is the number of e-folds since the cosmologically relevant modes exit the horizon till the end of inflation and χ is a dimensionless, slowly varying field

$$\chi = \frac{\varphi}{\sqrt{N} M_{Pl}} .$$

The slow-roll expansion becomes in this way a systematic $1/N$ expansion. The couplings in the inflaton Lagrangian become naturally small due to suppression factors arising from eq.(1.1) as the ratio of the two relevant energy scales here: M and M_{Pl} . The spectral index, the ratio of tensor/scalar fluctuations, the running index and the amplitude of the scalar adiabatic fluctuations are naturally

$$n_s - 1 = \mathcal{O}\left(\frac{1}{N}\right) , \quad r = \mathcal{O}\left(\frac{1}{N}\right) , \quad \frac{dn_s}{d \ln k} = \mathcal{O}\left(\frac{1}{N^2}\right) , \quad |\Delta_{k ad}^{\mathcal{R}}| \sim N \left(\frac{M}{M_{Pl}}\right)^2 , \quad (1.2)$$

for **all** inflaton potentials in the class of eq.(1.1). Hence, the energy scale of inflation M is completely determined by the amplitude of the scalar adiabatic fluctuations $|\Delta_{k ad}^{\mathcal{R}}|$ and using the WMAP5 results for it, we find $M \sim 10^{16}$ GeV. The running index results $dn_s/d \ln k \sim 1/N^2 \sim 10^{-4}$ and therefore can be neglected. Namely, within the class of models eq.(1.1) one does not need to measure the ratio r in order to learn about the scale of inflation. Moreover, we were able to provide **lower** bounds for r and **predict its value** in the effective theory of inflation using the CMB+LSS data and Monte Carlo Markov Chains (MCMC) simulations [5, 12].

- Besides its simplicity, the fourth order potential (minimal single field model in the Ginsburg-Landau spirit) is rich enough to describe the physics of inflation and accurately reproduce the WMAP data [5, 12]. It is well motivated within the Ginsburg-Landau approach as an effective field theory description (see ref.[14, 15]). We provided a complete analytic study complemented by a statistical analysis in [5, 12]. The MCMC analysis of the available CMB+LSS data with the Ginsburg-Landau effective field theory of inflation showed [5, 12]:

(i) The data strongly indicate a spontaneous **breaking** of the $\varphi \rightarrow -\varphi$ symmetry of the inflaton potentials. Namely, $w''(0) < 0$ is strongly favoured. Cubic terms do not improve the fit. (ii) Fourth order double-well potentials naturally satisfies this requirement and provide an excellent fit to the data. (iii) The above results and further physical analysis lead us to conclude that **new inflation** gives the best description of the data. (iv) We find a **lower bound** for r within fourth order double-well potentials: $r > 0.023$ (95% CL) and $r > 0.046$ (68% CL), see fig. 4. (v) The preferred new inflation potential is a double well, even function of the field with a moderate quartic coupling $y \sim 1$,

$$w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y} \right)^2 = -\frac{1}{2} \chi^2 + \frac{y}{32} \chi^4 + \frac{2}{y} . \quad (1.3)$$

[see eq.(1.1)]. This new inflation model yields as most probable values: $n_s \simeq 0.964$, $r \simeq 0.051$, see fig. 4. This value for r is within reach of forthcoming CMB observations [5, 12]. For the best fit value $y \simeq 1.26$, the inflaton field exits the horizon in the negative concavity region $w''(\chi) < 0$ intrinsic to new inflation. We find for the best fit, $M = 0.543 \times 10^{16}$ GeV for the scale of inflation and $m = 1.21 \times 10^{13}$ GeV for the inflaton mass. We derived explicit formulae and study in detail the spectral index n_s of the adiabatic fluctuations, the ratio r of tensor to scalar fluctuations and the running index $dn_s/d \ln k$ [5, 12]. We use these analytic formulas as hard constraints on n_s and r in the MCMC analysis. Our analysis differs in this **crucial** aspect from previous MCMC studies in the literature involving the CMB data.

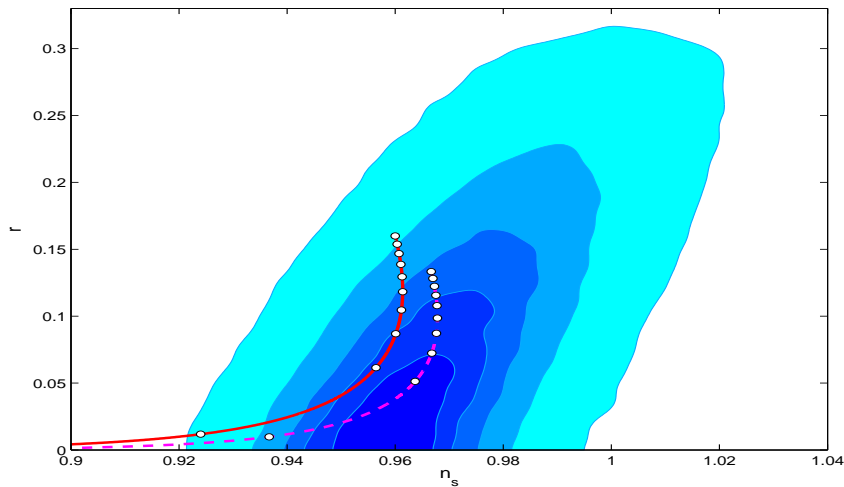


FIG. 4: (n_s, r) from the fourth order double-well inflaton potential eq.(1.3) compared to CMB+LSS data. The color-filled areas correspond to 12%, 27%, 45%, 68% and 95% confidence levels according to the WMAP, SN and Sloan data. The color of the areas goes from the darker to the lighter for increasing CL. The solid red curve is for $N = 50$ and the dashed magenta curve for $N = 60$. The quartic coupling y increases monotonically starting from the uppermost dots, corresponding to the free-field, purely quadratic inflaton potential $y = 0$ till the strong coupling region $y \gg 1$ in the lower left part of the curve. We see that very small values of r are excluded since they correspond to $n_s < 0.92$ outside the 95% confidence level contour. That is, we obtain a **lower bound** for r : $r > 0.027$ at 95% C. L.

- The effects of arbitrary **higher order** terms in the inflaton potential on the CMB observables: spectral index n_s and ratio r were systematically analyzed in ref. [44]. The theoretical values in the (n_s, r) plane for all double well inflaton potentials in the Ginsburg-Landau approach turn to be inside an **universal** banana-shaped region \mathcal{B} displayed in fig. 5. The upper border of the banana-shaped region \mathcal{B} is given by the fourth order double-well potential eq.(1.3) and provides an upper bound for the ratio r . The lower border of \mathcal{B} is defined by the quadratic plus an infinite barrier inflaton potential and provides a **lower bound** for the ratio r within the Ginsburg-Landau class of potentials [44]. For example, the current best value of the spectral index $n_s = 0.964$,

implies r is in the interval: $0.021 < r < 0.053$. Interestingly enough, this range is within reach of forthcoming CMB observations.

- The dynamics of inflation is usually described by the classical evolution of a scalar field (the inflaton). The use of classical dynamics is justified by the enormous stretching of physical lengths during inflation. When the physical wavelength of the fluctuations become larger than the Hubble radius, these fluctuations effectively become classical. This is probably the only case where the time evolution itself leads to the classicalization of fluctuations and microscopic scales near the Planck scale 10^{-32} cm $\lesssim \lambda = 2\pi/k \lesssim 10^{-28}$ cm become macroscopic today in the range $1 \text{ Mpc} \lesssim \lambda_{\text{today}} \lesssim 10^4 \text{ Mpc}$. This happens thanks to a redshift by $\sim 10^{56}$ since the beginning of inflation for a total number of inflation e-folds $N_{\text{tot}} \sim 64$.

The effective theory of inflation is generically valid as long as the energy density is $\ll M_{Pl}^4$ (sec. III). This is true thanks to eq.(1.1) even when the inflaton field φ takes values equal to many times M_{Pl} [5, 9].

We computed relevant quantum loop corrections to inflationary dynamics in ref. [5, 17, 18]. Novel phenomena emerge at the quantum level as a consequence of the lack of kinematic thresholds, among them the phenomenon of inflaton decay into its own quanta. A thorough study of the effect of quantum fluctuations reveals that these loop corrections are suppressed by powers of $(H/M_{Pl})^2$ where H is the Hubble parameter during inflation [5, 17, 18]. The amplitude of temperature fluctuations constrains the scale of inflation with the result that $(H/M_{Pl})^2 \sim 10^{-9}$. Therefore, quantum loop corrections are very small and controlled by the ratio $(H/M_{Pl})^2$, a conclusion that validates the reliability of the classical approximation and the effective field theory approach to inflationary dynamics. The quantum corrections to the power spectrum are computed and expressed in terms of n_s , r and $dn_s/d\ln k$. Trace anomalies dominate the quantum corrections to the primordial power spectrum (see [5, 17, 18]).

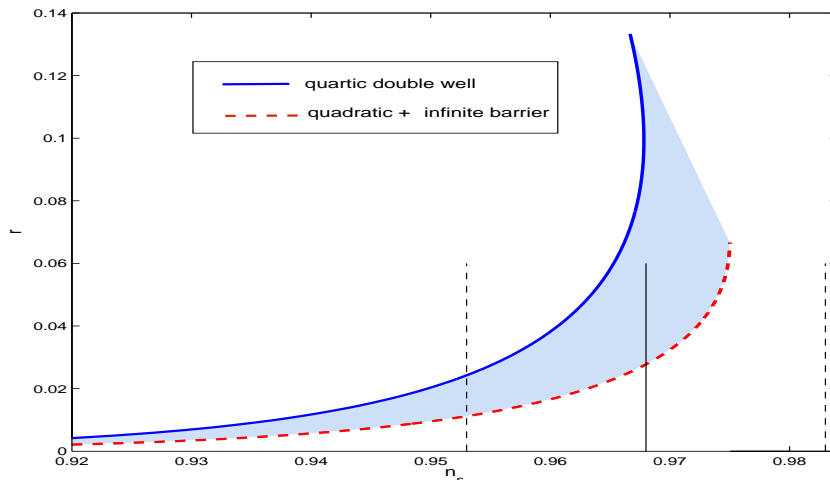


FIG. 5: We plot here the borders of the universal banana region \mathcal{B} in the (n_s, r) -plane setting $N = 60$. The curves are computed with the quadratic plus quartic potential eq.(1.3) and with the quadratic plus an infinite barrier inflaton potential [44].

- Scalar (curvature) and tensor (gravitational wave) perturbations originate in quantum fluctuations during inflation. These are usually studied within the slow-roll approximation and with Bunch-Davies initial conditions. We investigated the physical effects on the power spectrum of generic initial conditions with particular attention to back-reaction effects in refs. [19, 20]. We introduced a *transfer function* $D(k)$ which encodes the effect of generic initial conditions on the power spectra. The constraint from renormalizability and small back reaction entails that $D(k) \lesssim \mu^2/k^2$ for large k where μ characterizes the asymptotic decay of the occupation number. This implies that observable effects from initial conditions are more prominent in the *low* CMB multipoles. The effects on high l -multipoles are suppressed by a factor $\sim 1/l^2$ due to the large k fall off of $D(k)$. Hence, a change from the Bunch-Davies initial conditions for the fluctuations can naturally account for the low observed value of the CMB quadrupole [19–21, 45].
- The inflaton evolution generically starts by a fast-roll stage where the kinetic and potential energy of the inflaton are of the same order [5, 19, 20]. The universe expansion is non-inflationary to start (decelerated) and

hence no fluctuations leave the horizon then. This leads to a suppression of the quadrupole in curvature and tensor perturbations [5, 19–21]. The fast-roll stage becomes then accelerated (inflationary) and is followed by a slow-roll regime where the kinetic energy is much smaller than the potential energy. The slow-roll regime of inflation is an attractor of the dynamics [34] during which the Universe is dominated by vacuum energy. Inflation ends when again the kinetic energy of the inflaton becomes large as the field is rolling near the minimum of the potential. Eventually, the energy stored in the homogeneous inflaton is transferred explosively into the production of particles via spinodal or parametric instabilities [23–26]. More precisely, non-linear phenomena eventually **shut-off** the instabilities and **stop** inflation [23, 27, 28]. All these processes lead to the transition to a radiation dominated era. This is the standard picture of the transition from inflation to standard hot big bang cosmology.

- Within the context of the effective field theory and for generic initial conditions on the inflaton field, it is shown that a quadrupole suppression consistent with observations is a natural consequence of the fast-roll stage [19–21]. The fast-roll stage dynamically modifies the initial power spectrum of perturbations by a transfer function $D(k)$. We performed MCMC analysis of the WMAP and SDSS data combined with the most recent supernovae compilation [22] including the fast-roll stage. The quadrupole mode $k_Q = 0.238 \text{ Gpc}^{-1}$ exits the horizon about 0.2 efolds before the end of fast-roll. This fixes the redshift since the beginning of inflation till today to $z_{init} = 0.915 \times 10^{56}$. From that we find the **total number of efolds** N_{tot} during inflation to be (see ref. [5, 45]) $N_{tot} \simeq 64$ respecting the lower bound for N_{tot} that solves the horizon problem (see [5]). That is, the MCMC analysis of the CMB+LSS data including the early fast-roll explanation of the CMB quadrupole suppression **imposes** $N_{tot} \simeq 64$. Including the fast-roll stage improves the fits to the TT, the TE and the EE modes, well reproducing the quadrupole suppression.

We formulate inflation as an effective field theory within the Ginsburg-Landau spirit [5, 9, 14]. The theory of the second order phase transitions, the Ginsburg-Landau theory of superconductivity, the current-current Fermi theory of weak interactions, the sigma model of pions, nucleons (as skyrmions) and photons are all successful effective field theories. Our work shows how powerful is the effective theory of inflation **to predict observable quantities** that can be or will be soon contrasted with experiments. There are **two kind** of predictions in the effective theory of inflation: first, predictions on the order of magnitude of the CMB observables valid for all inflaton potentials in the class of eq.(1.1) as given by eqs.(1.2); second, precise quantitative predictions as those presented in refs. [5, 12, 21, 44, 45].

The Ginsburg-Landau realization of the inflationary potential fits the amplitude of the CMB anisotropy remarkably well and reveals that the Hubble parameter, the inflaton mass and non-linear couplings are see-saw-like, namely powers of the ratio $(M/M_{Pl})^2 \sim 10^{-9}$ multiplied by further powers of $1/N$ [5, 9]. Therefore, the smallness of the couplings is not a result of fine tuning but a **natural** consequence of the form of the potential, of the validity of the effective field theory description and slow-roll (sec. III A). The quantum expansion in loops is therefore a double expansion on $(H/M_{Pl})^2$ and $1/N$. Notice that graviton corrections are also of order $(H/M_{Pl})^2$ because the amplitude of tensor modes is of order H/M_{Pl} .

The infrared (superhorizon) modes in the quantum loops produce large contributions of the order $\sim N$. However, as shown in [5, 17, 18] these large infrared contributions get multiplied by slow-roll factors of order $\sim 1/N$. As a result, the superhorizon contributions to physical magnitudes turn to be of order N^0 [5, 17, 18] times factors of the order of $(H/M_{Pl})^2 \sim 10^{-9}$.

We note that the effective theory of inflation describes an evolution spanning about 26 orders of magnitude in length scales from the beginning till the end of the inflationary era. This is the largest scale change described by a field theory so far.

It must be stressed that the energy scale of inflation, $M \sim 10^{16} \text{ GeV}$ is the energy scale of at least two other important physical situations: (a) the scale of Grand Unification of strong and electroweak interactions and (b) the large energy scale in the see-saw formula for neutrino masses [see eq.(3.44)]. This coincidence suggests a physical link between the three areas.

Many deep problems remain to be solved in the early universe. One of them is the reheating problem. Namely, how the universe thermalizes after inflation and at which temperature. Baryogenesis provides a lower bound on the reheating temperature [2]. The mechanisms of thermalization uncovered in refs. [29] can provide a starting point to understand the reheating. The units used here are such that $\hbar = c = 1$.

II. THE STANDARD MODEL OF THE UNIVERSE

The history of the Universe is a history of expansion and cooling down.

On large scales the Universe is homogeneous and isotropic and its geometry is described by the spatially flat Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - a^2(t) d\vec{x}^2 \quad (2.1)$$

Overwhelming observational evidence indicates that the geometry of the Universe is spatially flat. Namely, in case the space is curved, its curvature radius is larger than the horizon and therefore inobservable.

Notice that this cosmological expansion has no center: it happens everywhere at all spatial points \vec{x} and it is identical everywhere. The scale factor grows monotonically with time.

Physical scales are stretched by the scale factor $a(t)$ with respect to the time independent comoving scales

$$l_{phys}(t) = a(t) l_{com} . \quad (2.2)$$

The redshift z at time t is defined as

$$z + 1 \equiv \frac{1}{a(t)} \quad (2.3)$$

where the scale factor today is chosen to be unit $a(0) \equiv 1$. The farther back in time, the larger is the redshift and the smaller is $a(t)$.

The temperature decreases as the universe expands as

$$T(t) = \frac{T_0}{a(t)} . \quad (2.4)$$

Eq.(2.4) applies to all particles in thermal equilibrium as well as to massless decoupled particles (radiation). Since the temperature decreased with time, the Universe underwent a succession of phase transitions towards the less symmetric phases [33].

The combination of data from CMB and LSS, and numerical simulations lead to the Λ CDM or *concordance model* which has now become the standard cosmology. This impressive convergence of observational data and theoretical and numerical results describes a Universe that is composed of a cosmological constant, dark matter, baryonic (atoms) matter and radiation. This model provides the **only consistent** explanation of the broad set of precise and independent astronomical observations over a wide range of scales available today. Namely:

- WMAP data and previous CMB data.
- Light Elements Abundances.
- Large Scale Structures (LSS) Observations. Baryon acoustic oscillations (BAO).
- Acceleration of the Universe expansion: Supernova Luminosity/Distance (SN) and Radio Galaxies.
- Gravitational Lensing Observations.
- Lyman α Forest Observations.
- Hubble Constant (H_0) Measurements.
- Properties of Clusters of Galaxies.
- Measurements of the Age of the Universe.

In the homogeneous and isotropic FRW universe described by eq.(2.1), the matter distribution must be homogeneous and isotropic, with an energy momentum tensor having in spatial average the isotropic fluid form

$$\langle T^\mu_\nu \rangle = \text{diag}[\rho, -p, -p, -p] , \quad (2.5)$$

where ρ , p are the energy density and pressure, respectively. In such space-time geometry the Einstein equations of general relativity reduce to the Friedmann equation, which determines the evolution of the scale factor from the energy density

$$\left[\frac{\dot{a}(t)}{a(t)} \right]^2 = H^2(t) = \frac{\rho}{3M_{Pl}^2} . \quad (2.6)$$

ρ_c	$(2.36 \text{ meV})^4$	h	0.705 ± 0.013
H_0	$h/[3 \text{ Gpc}] = h/[9.77813 \text{ Gyr}]$	Ω_Λ	0.726
M_{Pl}	$2.43534 \times 10^{18} \text{ GeV}$	Ω_M	0.274
M	$0.543 \times 10^{16} \text{ GeV}$	Ω_r	$8.49 \cdot 10^{-5}$
m	$1.21 \times 10^{13} \text{ GeV}$	n_s	0.960 ± 0.014

TABLE I: Selected Cosmological Parameters [8, 32]. m and M are given by eq.(3.42).

where $M_{Pl} = 1/\sqrt{8\pi G} = 2.43534 \times 10^{18} \text{ GeV} = 0.434 \times 10^{-5} \text{ g}$. The spatially flat Universe has today the critical density

$$\rho_c = 3 M_{Pl}^2 H_0^2 = 1.878 h^2 10^{-29} \text{ g/cm}^3 = 1.0537 10^{-5} h^2 \text{ GeV/cm}^3 = (2.518 \text{ meV})^4. \quad (2.7)$$

where $H_0 = 100 h \text{ km/sec/Mpc}$ is the Hubble constant today, $h = 0.705 \pm 0.013$ [8, 32] and then $H_0 = 1.5028 10^{-33} \text{ eV}$, $1 \text{ meV} = 10^{-3} \text{ eV}$. Notice that eq.(2.6) implies that $a(t)$ is a monotonic function of time.

The energy momentum tensor conservation reduces to the single conservation equation,

$$\dot{\rho} + 3 H(t) (\rho + p) = 0 \quad (2.8)$$

The two equations (2.6) and (2.8) can be combined to yield the acceleration of the scale factor,

$$\frac{\ddot{a}}{a} = -\frac{1}{6 M_{Pl}^2}(\rho + 3 p). \quad (2.9)$$

In order to provide a close set of equations we must append an equation of state $p = p(\rho)$ which is typically written in the form

$$p = w(\rho) \rho \quad (2.10)$$

The following are important cosmological solutions:

$$\text{Cosmological Constant} \Rightarrow p = -\rho : \mathbf{AD} \text{ de Sitter expansion} \Rightarrow \rho = \text{constant} ; a(t) = a(0) e^{Ht} ; H = \sqrt{\rho/[3 M_{Pl}^2]}$$

$$\text{Radiation} \Rightarrow p/\rho = 1/3 : \mathbf{RD} \text{ (Radiation domination)} \Rightarrow \rho(t) = \rho(t_r) a^{-4}(t) ; a(t) = a(t_r) \sqrt{t/t_r} \quad (2.11)$$

$$\text{Non-relativistic (cold) Matter} \Rightarrow p/\rho = 0 : \mathbf{MD} \text{ (Matter domination)} \Rightarrow \rho(t) = \rho(t_{eq}) a^{-3}(t) ; a(t) = a(t_{eq}) (t/t_{eq})^{\frac{2}{3}}$$

where t_r and t_{eq} are the values of cosmic time at which the Universe becomes radiation or matter dominated, respectively.

Notice from eq.(2.9) that accelerated expansion ($\ddot{a}(t) > 0$) takes place if $p/\rho < -1/3$.

The universe started by a very short accelerated inflationary stage dominated by the vacuum energy, lasting $\sim 10^{-36}$ sec ending by redshift $z \sim 10^{29}$ and approximately described by the de Sitter metric. This inflationary stage was followed by decelerated expansion, first by the radiation dominated era and then by the matter dominated era. Finally, the universe entered again an accelerated phase dominated by the dark energy, described by a cosmological constant in the Standard Model of the Universe, at $z \simeq 0.5$.

Particle physics at energy scales below $\sim 200 \text{ GeV}$ is on solid experimental footing in the framework of the standard model of strong and electroweak interactions.

Current theoretical ideas supported by the renormalization group running of the couplings in the standard model of particle physics and its supersymmetric extensions show that the strong, weak and electromagnetic interactions are unified in a grand unified theory (GUT) at the scale $M_{GUT} \sim 10^{16} \text{ GeV}$. Furthermore, the characteristic scale at which gravity calls for a quantum description is the Planck scale $M_{Pl} = 1/\sqrt{8\pi G} = 2.43534 10^{18} \text{ GeV} \gg M_{GUT}$.

The connection between the standard model of particle physics and early Universe cosmology is through the semiclassical Einstein equations that couple the space-time geometry to the matter-energy content. As argued above, gravity can be studied semi-classically at energy scales well below the Planck scale. The standard model of particle physics is a *quantum field theory*, thus the space-time is classical but with sources that are quantum fields. Semiclassical gravity is defined by the Einstein's equations with the expectation value of the quantum energy-momentum tensor $\hat{T}^{\mu\nu}$ as the source

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \frac{\langle \hat{T}^{\mu\nu} \rangle}{M_{Pl}^2}. \quad (2.12)$$

The expectation value of $\hat{T}^{\mu\nu}$ is taken in a given quantum state (or density matrix) compatible with homogeneity and isotropy which must be translational and rotational invariant. Such state yields an expectation value for the energy momentum tensor with the fluid form eq.(2.5), and the Einstein equations (2.12) reduce to the Friedmann equation (2.6).

All of the ingredients are now in place to understand the evolution of the early Universe. Einstein's equations determine the evolution of the scale factor, particle physics provides the energy momentum tensor and statistical mechanics provides the fundamental framework to describe the thermodynamics from the microscopic quantum field theory of the strong, electroweak interactions and beyond.

The sources for Einstein equations are dark energy, dark and ordinary matter and radiation. The standard model of particle physics describes ordinary matter and radiation.

Dark energy accounts **today** for $72 \pm 1.5\%$ of the energy of the Universe [8]. The current observed value is $\rho_\Lambda = \Omega_\Lambda \rho_c = (2.36 \text{ meV})^4$ from eq.(2.7) [8, 32]. The equation of state is $p_\Lambda = -\rho_\Lambda$ within observational errors corresponding to a cosmological constant.

The nature of the dark energy (today) is not yet understood. A plausible explanation of the dark energy may be the quantum zero-point energy of a light matter field in the cosmological space-time. This has the equation of state of a cosmological constant. Notice that the renormalized value of the zero point energy in the cosmological space-time is finite and may be naturally of the order of the $(\text{mass})^4$ of the light field involved.

Matter accounts today for $28 \pm 1.5\%$ of the energy of the Universe [8]. 84% of the matter is **dark matter**. Therefore, dark matter is an essential constituent of the universe. The nature of dark matter is still unknown but is certainly beyond the Standard Model of strong and electroweak particle interactions [3, 33]. It is probably formed by particles in the keV mass scale as discussed in sec. IV [42, 43].

Main events in the universe after inflation are (see fig. 6):

- Begining of the **RD** era and end of inflation: $z \sim 10^{29}$, $T_{reh} \sim 10^{16} \text{ GeV}$, $t \sim 10^{-36} \text{ sec}$.
- Electro-Weak phase transition: $z \sim 10^{15}$, $T_{EW} \sim 100 \text{ GeV}$, $t \sim 10^{-11} \text{ sec}$.
- QCD phase transition (confinement): $z \sim 10^{12}$, $T_{QCD} \sim 170 \text{ MeV}$, $t \sim 10^{-5} \text{ sec}$.
- Big bang nucleosynthesis (BBN): $z \sim 10^9$, $\ln(1+z) \sim 21$, $T \simeq 0.1 \text{ MeV}$, $t \sim 20 \text{ sec}$.
- Radiation-Matter equality: $z \simeq 3200$, $\ln(1+z) \simeq 8$, $T \simeq 0.7 \text{ eV}$, $t \sim 57000 \text{ yr}$.
- CMB last scattering: $z \simeq 1100$, $\ln(1+z) \simeq 7$, $T \simeq 0.25 \text{ eV}$, $t \sim 370000 \text{ yr}$.
- Matter-Dark Energy equality: $z \simeq 0.47$, $\ln(1+z) \simeq 0.38$, $T \simeq 0.345 \text{ meV}$, $t \sim 8.9 \text{ Gyr}$.
- Today: $z = 0$, $\ln(1+z) = 0$, $T = 2.725 \text{ K} = 0.2348 \text{ meV}$, $t \equiv t_0 = 13.72 \text{ Gyr}$.

In fig. 6 we plot ρ_Λ/ρ , ρ_{Matter}/ρ and $\rho_{radiation}/\rho$ as functions of $\log(1+z)$ where $\rho_\Lambda = \Lambda$, $\rho_{Matter} = \Omega_M/a^3$ and $\rho_{radiation} = \Omega_r/a^4$. Notice that $\rho_\Lambda + \rho_{Matter} + \rho_{radiation} = \rho$.

In summary, the Friedmann equation (2.6) can be written as

$$H^2(t) = H_0^2 \left[\Omega_\Lambda + \frac{\Omega_M}{a^3} + \frac{\Omega_r}{a^4} \right]. \quad (2.13)$$

The temperature of the universe in the post-inflation radiation dominated era (reheating temperature T_r) is bounded from below in order to explain the baryon asymmetry and the Big Bang Nucleosynthesis (BBN). This amounts to a further constraint on the inflationary model. The BBN constraint is the milder. If the observed baryon asymmetry is produced at the electroweak scale, the constraint on the reheating temperature is $\gtrsim 100 \text{ GeV}$, however the origin of the baryon asymmetry may be at the GUT scale in which case the reheating temperature should be $T_r > 10^9 \text{ GeV}$ [2].

III. INFLATION AND INFLATON FIELD DYNAMICS

A simple implementation of the inflationary scenario is based on a single scalar field, the *inflaton* with a Lagrangian density

$$\mathcal{L} = a^3(t) \left[\frac{\dot{\varphi}^2}{2} - \frac{(\nabla\varphi)^2}{2a^2(t)} - V(\varphi) \right], \quad (3.1)$$

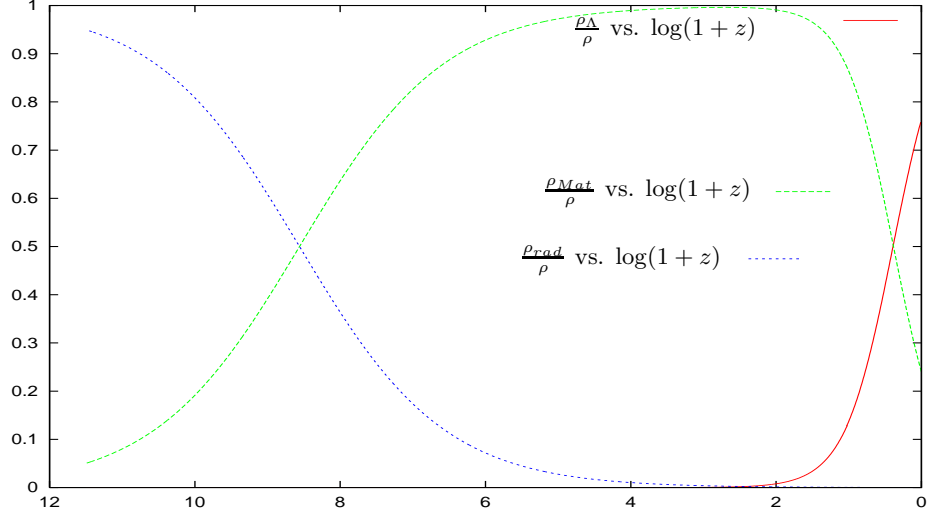


FIG. 6: $\frac{\rho_\Lambda}{\rho}$, $\frac{\rho_{Matter}}{\rho}$ and $\frac{\rho_{radiation}}{\rho}$ vs. $\log(1+z)$.

where $V(\varphi)$ is the inflaton potential. Since the universe expands exponentially fast during inflation, gradient terms are exponentially suppressed and can be neglected. At the same time, the exponential stretching of spatial lengths classicalize the physics and permits a classical treatment. One can therefore consider an homogeneous and classical inflaton field $\varphi(t)$ which obeys the evolution equation

$$\ddot{\varphi} + 3H(t)\dot{\varphi} + V'(\varphi) = 0. \quad (3.2)$$

in the isotropic and homogeneous FRW metric eq.(2.1) which is sourced by the inflaton according to the Friedmann equation eq.(2.6). Eq.(2.6) takes here the form

$$H^2(t) = \frac{1}{3M_{Pl}^2} \left[\frac{1}{2} \dot{\varphi}^2 + V(\varphi) \right]. \quad (3.3)$$

The energy density and the pressure for a spatially homogeneous inflaton are given by

$$\rho = \frac{\dot{\varphi}^2}{2} + V(\varphi) \quad , \quad p = \frac{\dot{\varphi}^2}{2} - V(\varphi). \quad (3.4)$$

The time derivative of the Hubble parameter takes the form

$$\dot{H}(t) = -\frac{\dot{\varphi}^2}{2M_{Pl}^2} \quad (3.5)$$

where we used eqs.(3.2) and (3.3). This shows that $H(t)$ **decreases monotonically** with time.

The inflaton fields starts with some chosen values of φ and $\dot{\varphi}$ and evolves together with the scale factor according to eqs.(3.2) and (3.3). The inflaton clearly rolls down the slope of the potential going towards a local minimum of $V(\varphi)$. The basic constraint on the inflationary potential is

$$V(\varphi_{min}) = V'(\varphi_{min}) = 0. \quad (3.6)$$

That is, the inflaton potential **must vanish** at its minimum φ_{min} in order to have a finite number of e-folds. The inflaton evolves from its initial value towards the minimum φ_{min} . If $V(\varphi_{min}) > 0$, we see from eq.(3.3) that inflation will be eternal. That is, a de Sitter phase will continue forever with the inflaton at the constant value φ_{min} .

There are two main classes of inflaton potentials leading to two main classes of inflation.

(a) In **small field inflation** the minimum of the potential is at a non-zero value $\varphi_{min} \neq 0$ and the inflaton field starts near (or at) $\varphi = 0$ evolving towards $\varphi = \varphi_{min}$. These are discrete symmetry ($\varphi \rightarrow -\varphi$) breaking potentials [30],

$$V(\varphi) = \frac{\lambda}{4} \left(\varphi^2 - \frac{m^2}{\lambda} \right)^2 = -\frac{m^2}{2} \varphi^2 + \frac{\lambda}{4} \varphi^4 + \frac{m^4}{4\lambda} \quad , \quad \text{new inflation} . \quad (3.7)$$

(b) In **large field inflation** the minimum of the potential is at $\varphi_{min} = 0$ and the inflaton field starts at $\varphi \gg M$ evolving towards $\varphi = 0$. These are unbroken symmetry potentials [31],

$$V(\varphi) = +\frac{m^2}{2} \varphi^2 + \frac{\lambda}{4} \varphi^4 \quad , \quad \text{chaotic inflation} . \quad (3.8)$$

For historical reasons large field inflation is often called **chaotic inflation** and small field inflation **new inflation**.

Inflation should last at least $N_{tot} \gtrsim 64$ efolds in order to solve the entropy, horizon and flatness problems (see, for example, ref. [5]). Inflation can produce such large number of efolds provided it lasts enough time. This can be achieved if the inflaton evolves slowly (slow-roll), namely $\dot{\varphi}^2 \ll V(\varphi)$. This implies from eq.(3.4) that

$$\rho = -p \simeq V(\varphi) \simeq \text{constant},$$

as the equation of state leading to a de Sitter universe. Eq.(3.3) yields as scale factor

$$a(t) \simeq e^{H t} \quad , \quad H \simeq \sqrt{\frac{V(\varphi)}{3 M_{Pl}^2}} \quad (3.9)$$

[see eq.(2.11)]. However, eq.(3.9) is only an approximation to the slow-roll inflationary dynamics. The scale factor in the slow-roll approximation is presented in ref. [5, 45].

While inflationary dynamics is typically studied in terms of a *classical* homogeneous inflaton field as explained above, such classical field must be understood as the expectation value of a *quantum field* in an isotropic and homogeneous quantum state.

In ref.[23, 27, 28] the *quantum dynamics* of inflation was studied for small and large field inflation. The initial quantum state was taken to be a gaussian wave function(al) with vanishing or non-vanishing expectation value of the field. This state evolves in time with the full inflationary potential which features an unstable (spinodal) region for $\varphi^2 < m^2/(3 \lambda)$ where $V''(\varphi) < 0$ in the broken symmetric case eq.(3.7). Just as in the case of Minkowski space time, there is a band of spinodally or parametrically unstable wavevectors, in which the amplitude of the quantum fluctuations grows exponentially fast [23, 33]. Because of the cosmological expansion wave vectors are redshifted into the unstable band and when the wavelength of the unstable modes becomes larger than the Hubble radius these modes become *classical* with a large amplitude and a frozen phase. These long wavelength modes assemble into a classical coherent and homogeneous condensate, which obeys the equations of motion of the classical inflaton[23, 27, 28]. This phenomenon of classicalization and the formation of a homogeneous condensate takes place during the *first* 5 – 10 efolds after the beginning of inflation. The **non perturbative** quantum field theory treatment in refs.[23, 27, 28] shows that this rapid redshift and classicalization justifies the use of an homogeneous classical inflaton leading to the following robust conclusions [23, 27, 28]:

- The quantum fluctuations of the inflaton are of two different kinds: (a) Large amplitude quantum inflaton fluctuations generated at the beginning of inflation through spinodal instabilities or parametric resonance depending on the inflationary scenario chosen. They have at the beginning of inflation physical wavenumbers in the range of [23, 27, 28]

$$k \lesssim 10 m \quad , \quad (3.10)$$

and they become superhorizon a few efolds after the beginning of inflation. The phase of these long-wavelength inflaton fluctuations freeze out and their amplitude grows thereby effectively forming a homogeneous *classical* inflaton condensate. The study of more general initial quantum states featuring highly excited distribution of quanta lead to similar conclusions [28]: during the first few efolds of evolution the rapid redshift produces a classicalization of long-wavelength inflaton fluctuations and the emergence of a homogeneous coherent inflaton condensate obeying the *classical equations of motion* in terms of the inflaton potential. (b) Cosmological scales relevant for the observations *today* between ~ 1 Mpc and the horizon today had exited the Hubble radius inside a window of about 10 e-folds, from ~ 63 to ~ 53 efolds before the end of inflation [2]. These correspond to small fluctuations of high physical wavenumbers at the beginning of inflation in the range given by [5]

$$3.8 \cdot 10^{14} \text{ GeV } \beta^{-1} e^{N_{tot}-64} < k^{init} < 3.8 \cdot 10^{18} \text{ GeV } \beta^{-1} e^{N_{tot}-64} . \quad (3.11)$$

where $\beta \equiv \sqrt{10^{-4} M_{Pl}/H} \sim 1$ and H is Hubble by the end of inflation. Since [eq.(3.40)] $m \sim 10^{13}$ GeV, we see that the large amplitude modes eq.(3.10) for typical values $N_{tot} \sim 64$ and $H \sim 10^{-4} M_{Pl}$ correspond to scales **larger** than the horizon today.

- During the rest of the inflationary stage the dynamics is described by this classical homogeneous condensate that obeys the classical equations of motion with the inflaton potential. Thus, inflation even if triggered by an initial quantum state or density matrix of the quantum field, is effectively described in terms of a classical homogeneous scalar condensate.

The body of results emerging from these studies provide a justification for the description of inflationary dynamics in terms of a *classical* homogeneous scalar field. The conclusion is that after a few initial e-folds during which the unstable wavevectors are redshifted well beyond the Hubble radius, all what remains for the ensuing dynamics is a homogeneous classical condensate, plus small quantum fluctuations corresponding to the wave k -modes.

These **small** quantum fluctuations include scalar curvature and tensor gravitational fluctuations. They must be treated together with the inflaton fluctuations in the unified gauge invariant approach [4, 5]. In the treatment of **large** amplitude quantum inflaton fluctuations, gravitational fluctuations can be safely neglected [23].

Inflation based on a scalar inflaton field should be considered as an **effective theory**, namely, not necessarily a fundamental theory but as a low energy limit of a microscopic fundamental theory. The inflaton may be a coarse-grained average of fundamental scalar fields, or a composite (bound state) of fields with higher spin, just as in superconductivity. Bosonic fields do not need to be fundamental fields, for example they may emerge as condensates of fermion-antifermion pairs $\langle \bar{\Psi}\Psi \rangle$ in a grand unified theory (GUT) in the cosmological background. In order to describe the cosmological evolution it is enough to consider the effective dynamics of such condensates.

We computed in close form the inflaton potential dynamically generated when the inflaton field is a fermion condensate in the inflationary universe [44]. We considered in ref. [44] the inflaton field coupled to Dirac fermions Ψ through the interaction Lagrangian

$$\mathcal{L} = \bar{\Psi} [i \gamma^\mu \mathcal{D}_\mu - m_f - g_Y \varphi] \Psi , \quad (3.12)$$

Where g_Y stands for a generic Yukawa coupling and the fermion mass m_f is absorbed by a constant shift of the inflaton field. The γ^μ are Dirac γ -matrices in curved space-time and \mathcal{D}_μ stands for the fermionic covariant derivative.

The effective potential of fermions can be computed in de Sitter inflation with the result [5, 18],

$$V_f(\varphi) = V_0 - \frac{1}{2} m^2 \varphi^2 + \frac{1}{4} \lambda \varphi^4 + H^4 Q \left(g_Y \frac{\varphi}{H} \right) , \quad (3.13)$$

where,

$$Q(x) = \frac{x^2}{8\pi^2} \left\{ (1+x^2) [\gamma + \text{Re} \psi(1+ix)] - \zeta(3) x^2 \right\} , \quad x \equiv g_Y \frac{\varphi}{H} . \quad (3.14)$$

The constant V_0 must be such that $V_f(\varphi)$ fulfills eq.(3.6), $m^2 > 0$ and λ are the renormalized mass and renormalized coupling constant, $\psi(x)$ stands for the digamma function, γ for the Euler-Mascheroni constant and $\zeta(x)$ for the Riemann zeta function. The inflaton potential $V_f(\varphi)$ turns to belong to the Ginsburg-Landau class and provides (n_s, r) inside the **universal** banana-shaped region \mathcal{B} depicted in fig. 5 [44].

The relation between the low energy effective field theory of inflation and the microscopic fundamental theory is akin to the relation between the effective Ginsburg-Landau theory of superconductivity [14] and the microscopic BCS theory, or like the relation of the $O(4)$ sigma model, an effective low energy theory of pions, photons and nucleons (as skyrmions), with quantum chromodynamics (QCD) [15]. The guiding principle to construct the effective theory is to include the appropriate symmetries [15]. Contrary to the sigma model where the chiral symmetry strongly constraints the model [15], only general covariance can be imposed on the inflaton model.

In summary, the physics during inflation is characterized by:

- Out of equilibrium matter field evolution in a rapidly expanding space-time dominated by the vacuum energy. The scale factor is quasi-de Sitter: $a(t) \simeq e^{Ht}$.
- Extremely high energy density at the scale of $\lesssim 10^{16}$ GeV.
- Explosive particle production at the beginning of inflation due to spinodal or parametric **instabilities** for new and chaotic inflation, respectively [23, 27].
- The enormous redshift as a consequence of a large number of e-folds (~ 64) classicalizes the dynamics: an **assembly** of (superhorizon) fluctuations behave as the classical and homogeneous inflaton field. The inflaton which is a long-wavelength condensate slowly rolls down the potential hill towards its minimum [27].

- Quantum non-linear phenomena eventually **shut-off** the instabilities and **stop** inflation [23, 27, 28].

As indicated above eq.(3.11), the cosmologically relevant fluctuations have at the beginning of inflation physical wavelengths in a range reaching the Planck scale

$$3.3 \cdot 10^{-32} e^{64-N_{tot}} \beta \text{ cm} \lesssim \lambda^{init} = 2\pi/k^{init} \lesssim 3.3 \cdot 10^{-28} e^{64-N_{tot}} \beta \text{ cm} ,$$

These fluctuations become macroscopic through the huge redshift during inflation and the subsequent expansion of the universe with wavelengths today in the range $1 \text{ Mpc} \lesssim \lambda_{today} \lesssim 10^4 \text{ Mpc}$. Namely, a total redshift of 10^{56} . During this process these quantum fluctuations classicalize just due to the huge stretching of the lengths. A field theoretical treatment shows that the quantum density matrix of the inflaton becomes diagonal in the inflaton field representation as inflation ends [27].

A. Slow-roll, the Universal Form of the Inflaton Potential and the Energy Scale of Inflation

The inflaton potential $V(\varphi)$ must be a slowly varying function of φ in order to permit a slow-roll solution for the inflaton field $\varphi(t)$ which guarantees a total number of efolds ~ 64 as discussed in [5]. Slow-roll inflation corresponds to a fairly flat potential and the slow-roll approximation usually invokes a hierarchy of dimensionless ratios in terms of the derivatives of the potential [3, 4]. We recasted the slow-roll approximation as an expansion in $1/N$ where $N \sim 60$ is the number of efolds since the cosmologically relevant modes exit the horizon till the end of inflation [9].

In the slow-roll regime higher time derivatives can be neglected in the evolution eqs.(3.2) and (3.3) with the result

$$3 H(t) \dot{\varphi} + V'(\varphi) = 0 \quad , \quad H^2(t) = \frac{V(\varphi)}{3M_{Pl}^2} \quad (3.15)$$

These first order equations can be solved in closed form as

$$M_{Pl}^2 N[\varphi] = - \int_{\varphi}^{\varphi_{end}} V(\varphi) \frac{d\psi}{dV} d\psi \quad . \quad (3.16)$$

where $N[\varphi]$ is the number of e-folds since the field φ exits the horizon till the end of inflation (where φ takes the value φ_{end}). This is in fact the slow roll solution of the evolution equations eqs.(3.2) and (3.3) in terms of quadratures.

Eq.(3.16) indicates that $M_{Pl}^2 N[\varphi]$ scales as φ^2 and therefore the field φ is of the order $\sqrt{N} M_{Pl} \sim \sqrt{60} M_{Pl}$ for the cosmologically relevant modes. Therefore, we propose as universal form for the inflaton potential [9]

$$V(\varphi) = N M^4 w(\chi) \quad , \quad (3.17)$$

where χ is a dimensionless, slowly varying field

$$\chi = \frac{\varphi}{\sqrt{N} M_{Pl}} \quad , \quad (3.18)$$

More precisely, we choose $N \equiv N[\varphi]$ as the number of e-folds since a pivot mode k_0 exits the horizon till the end of inflation. Eq.(3.17) includes all well known slow-roll families of inflation models such as new inflation [30], chaotic inflation [31], natural inflation [39], etc.

The dynamics of the rescaled field χ exhibits the slow time evolution in terms of the *stretched* dimensionless cosmic time variable,

$$\tau = \frac{t M^2}{M_{Pl} \sqrt{N}} \quad , \quad \mathcal{H} \equiv \frac{H M_{Pl}}{\sqrt{N} M^2} = \mathcal{O}(1) \quad . \quad (3.19)$$

The rescaled variables χ and τ change slowly with time. Eq.(3.18) shows that a large change in the field amplitude φ results in a small change in the χ amplitude; a change in $\varphi \sim M_{Pl}$ results in a χ change $\sim 1/\sqrt{N}$. The form of the potential, eq.(3.17), the rescaled dimensionless inflaton field eq.(3.18) and the time variable τ make **manifest** the slow-roll expansion as a consistent systematic expansion in powers of $1/N$ [9].

We can choose $|w''(0)| = 1$ without losing generality. Then, the inflaton mass scale around zero field is given by a see-saw formula

$$m^2 = |V''(\varphi=0)| = \frac{M^4}{M_{Pl}^2} \quad , \quad m = \frac{M^2}{M_{Pl}} \quad . \quad (3.20)$$

The Hubble parameter when the cosmologically relevant modes exit the horizon is given by

$$H = \sqrt{N} m \mathcal{H} \sim 7 m , \quad (3.21)$$

where we used that $\mathcal{H} \sim 1$. As a result, $m \ll M$ and $H \ll M_{Pl}$. The value of M is determined by the amplitude of the CMB fluctuations within the effective theory of inflation. We obtain in sec. IIIB [see eqs.(3.39) and (3.40)]: $M \sim 0.70 \cdot 10^{16}$ GeV, $m \sim 2.04 \cdot 10^{13}$ GeV and $H \sim 10^{14}$ GeV for generic slow-roll potentials eq.(3.17).

The energy density and the pressure [eq.(3.4)] in terms of the dimensionless rescaled field χ and the slow time variable τ take the form,

$$\frac{\rho}{N M^4} = \frac{1}{2 N} \left(\frac{d\chi}{d\tau} \right)^2 + w(\chi) \quad , \quad \frac{p}{N M^4} = \frac{1}{2 N} \left(\frac{d\chi}{d\tau} \right)^2 - w(\chi) . \quad (3.22)$$

The equations of motion (3.2) and (3.3), in the same variables become

$$\begin{aligned} \mathcal{H}^2(\tau) &= \frac{\rho}{N M^4} = \frac{1}{3} \left[\frac{1}{2 N} \left(\frac{d\chi}{d\tau} \right)^2 + w(\chi) \right] , \\ \frac{1}{N} \frac{d^2\chi}{d\tau^2} + 3 \mathcal{H} \frac{d\chi}{d\tau} + w'(\chi) &= 0 . \end{aligned} \quad (3.23)$$

The slow-roll approximation follows by neglecting the $1/N$ terms in eqs.(3.23). Both $w(\chi)$ and $\mathcal{H}(\tau)$ are of order N^0 for large N . Both equations make manifest the slow-roll expansion as an expansion in $1/N$.

Eq.(3.16) in terms of the field χ takes the form

$$- \int_{\chi_{exit}}^{\chi_{end}} \frac{w(\chi)}{w'(\chi)} d\chi = 1 . \quad (3.24)$$

This gives $\chi = \chi_{exit}$ at horizon exit as a function of the couplings in the inflaton potential $w(\chi)$.

Inflation ends after a finite number of efolds provided [see eq.(3.6)]

$$w(\chi_{end}) = w'(\chi_{end}) = 0 . \quad (3.25)$$

So, this condition is enforced in the inflationary potentials.

For the quartic degree potentials $V(\varphi)$ eqs.(3.7)-(3.8), the corresponding dimensionless potentials $w(\chi)$ take the form

$$w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y} \right)^2 = -\frac{1}{2} \chi^2 + \frac{y}{32} \chi^4 + \frac{2}{y} , \quad \text{new inflation} , \quad (3.26)$$

$$w(\chi) = \frac{1}{2} \chi^2 + \frac{y}{32} \chi^4 , \quad \text{chaotic inflation} , \quad (3.27)$$

where the coupling y is of **order one** and

$$\lambda = \frac{y}{8 N} \left(\frac{M}{M_{Pl}} \right)^4 \ll 1 \quad \text{since} \quad M \ll M_{Pl} .$$

For a general potential $V(\varphi)$ we can always eliminate the linear term by a shift in the field φ without losing generality,

$$V(\varphi) = V_0 \pm \frac{1}{2} m^2 \varphi^2 + \sum_{n=3}^{\infty} \frac{\lambda_n}{n} \varphi^n , \quad (3.28)$$

and

$$w(\chi) = w_0 \pm \frac{1}{2} \chi^2 + \sum_{n=3}^{\infty} \frac{G_n}{n} \chi^n , \quad (3.29)$$

where the dimensionless coefficients G_n are of order one. We find from eqs. (3.17) and (3.18),

$$V_0 = N M^4 w_0 \quad , \quad \lambda_n = \frac{G_n M^4}{N^{\frac{n}{2}-1} M_{Pl}^n} , \quad (3.30)$$

In particular, we get comparing with eqs.(3.7), (3.8), (3.26) and (3.27),

$$\lambda_3 = \frac{G_3}{\sqrt{N}} \frac{M^4}{M_{Pl}^3} \quad , \quad \lambda = \lambda_4 = \frac{G_4}{N} \left(\frac{M}{M_{Pl}} \right)^4 \quad , \quad G_4 = \frac{y}{8} . \quad (3.31)$$

We find the dimensionful couplings λ_n suppressed by the n th power of M_{Pl} as well as by the factor $N^{\frac{n}{2}-1}$. Notice that this suppression factors are natural and come from the ratio of the two relevant energy scales here: the Planck mass and the inflation scale M .

In new inflation with the potential of eq.(3.26), the inflaton starts near the local maximum $\chi = 0$ and keeps rolling down the potential hill till it reaches the absolute minimum $\chi = \sqrt{8/y}$. The initial values of χ and $\dot{\chi}$ must be chosen to have a total of $\gtrsim 64$ efolds of inflation. In all cases $\chi(0)$ and $\dot{\chi}(0)$ turn to be of order one.

There are two *generic* inflationary regimes: slow-roll and fast-roll depending on whether

$$\begin{aligned} \frac{1}{2N} \left(\frac{d\chi}{d\tau} \right)^2 &\ll w(\chi) & : & \text{slow-roll regime} \\ \frac{1}{2N} \left(\frac{d\chi}{d\tau} \right)^2 &\sim w(\chi) & : & \text{fast-roll regime} \end{aligned} \quad (3.32)$$

Both regimes show up in **all** inflationary models in the class eq.(3.17). A fast-roll stage emerges from generic initial conditions for the inflaton field. This fast-roll stage is generally very short and is followed by the slow-roll stage (see sec. [5, 45]). The slow-roll regime is an attractor in this dynamical system [34].

Eq.(3.17) for the inflaton potential resembles the moduli potential coming from supersymmetry breaking,

$$V_{susy}(\varphi) = m_{susy}^4 v \left(\frac{\varphi}{M_{Pl}} \right) , \quad (3.33)$$

where m_{susy} stands for the supersymmetry breaking scale. In our context, eq.(3.33) implies that $m_{susy} \sim 10^{16}$ Gev. That is, the supersymmetry breaking scale m_{susy} turns out to be at the GUT scale $m_{susy} \sim M_{GUT}$.

It must be stressed that the validity of the inflaton potential eq.(3.17) is independent of whether or not there is an underlying supersymmetry. In addition, the observational support on inflaton potentials like eq.(3.17) can be taken as a first signal of the presence of supersymmetry in a cosmological context. No experimental signals of supersymmetry are known so far despite the enormous theoretical work done on supersymmetry since 1971.

B. The energy scale of inflation and the quasi-scale invariance during inflation.

The inflationary scalar power spectrum can be written as [4, 5]

$$P_{\mathcal{R}}^{BD}(k) = |\Delta_{k\,ad}^{\mathcal{R}}|^2 \left(\frac{k}{k_0} \right)^{n_s-1} , \quad (3.34)$$

where in the slow-roll approximation (leading order in $1/N$) the amplitude $|\Delta_{k\,ad}^{\mathcal{R}}|^2$ takes the form [5]

$$|\Delta_{k\,ad}^{\mathcal{R}}|^2 = \frac{N^2}{12\pi^2} \left(\frac{M}{M_{Pl}} \right)^4 \frac{w^3(\chi)}{w'^2(\chi)} . \quad (3.35)$$

where $\chi \equiv \chi_{exit}$ stands for the inflaton field at horizon exit and n_s stands for the spectral index

$$n_s - 1 = -\frac{3}{N} \left[\frac{w'(\chi)}{w(\chi)} \right]^2 + \frac{2}{N} \frac{w''(\chi)}{w(\chi)} + \mathcal{O} \left(\frac{1}{N^2} \right) . \quad (3.36)$$

Since, $w(\chi)$ and $w'(\chi)$ are of order one, we find from eq.(3.35)

$$\left(\frac{M}{M_{Pl}} \right)^2 \sim \frac{2\sqrt{3}\pi}{N} |\Delta_{k\,ad}^{\mathcal{R}}| \simeq 0.897 \times 10^{-5} . \quad (3.37)$$

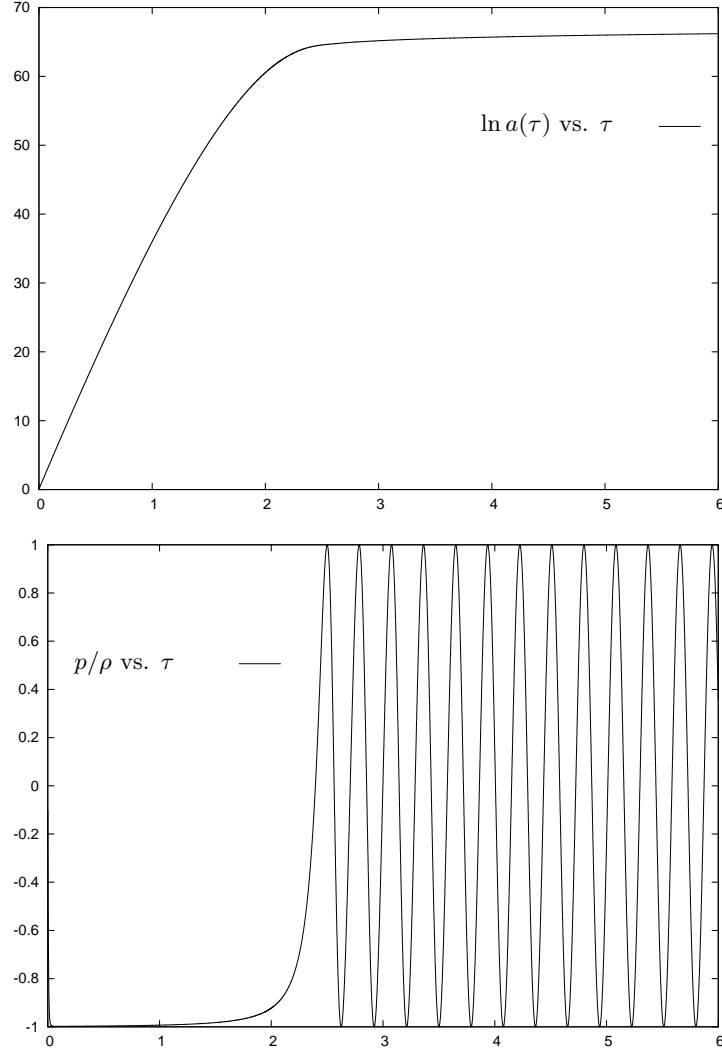


FIG. 7: Upper panel: $\ln a(\tau)$ which is the number of e-folds as a function of the stretched cosmic time eq.(3.19). We see that $a(\tau)$ grows exponentially with time (quasi-de Sitter inflation) for $\tau < \tau_{end} \simeq 2.39$. Lower panel: the equation of state p/ρ vs. τ [eq.(2.10)]. We have $p/\rho \simeq -1$ after the fast-roll stage and before the end of inflation. That is, for $0.0247 \lesssim \tau < \tau_{end} \simeq 2.39$, p/ρ oscillates with zero average: $\langle p/\rho \rangle = 0$ during the subsequent matter dominated era. Both figures are for the inflaton potential eq.(3.26) with $y = 1.26$ and $N_{tot} = 64$ efolds of inflation.

where we used $N \simeq 60$, set $k = k_0$ with $k_0 = 0.002 \text{ (Mpc)}^{-1}$ the WMAP pivot scale and ref.[8]

$$|\Delta_{k_{ad}}^{\mathcal{R}}| = (4.94 \pm 0.1) \times 10^{-5} . \quad (3.38)$$

This fixes the scale of inflation to be

$$M \sim 2.99 \times 10^{-3} M_{Pl} \sim 0.73 \times 10^{16} \text{ GeV} . \quad (3.39)$$

This value *pinpoints the scale of the potential* during inflation to be at the GUT scale suggesting a deep connection between inflation and the physics at the GUT scale in cosmological space-time.

As a consequence we get for the inflaton mass and the Hubble parameter during inflation from eq.(3.20)-(3.21),

$$m = \frac{M^2}{M_{Pl}} \sim 2.18 \times 10^{13} \text{ GeV} \quad , \quad H \sim 10^{14} \text{ GeV} \quad (3.40)$$

Notice that these values for the inflation scale M and the inflaton mass are **model independent** within the slow-roll class of models eq.(3.17). In addition, we see that $m \simeq 0.003 M$. Namely, the inflaton is a **very light** field in this context. We can therefore expect infrared and scale invariant phenomena here.

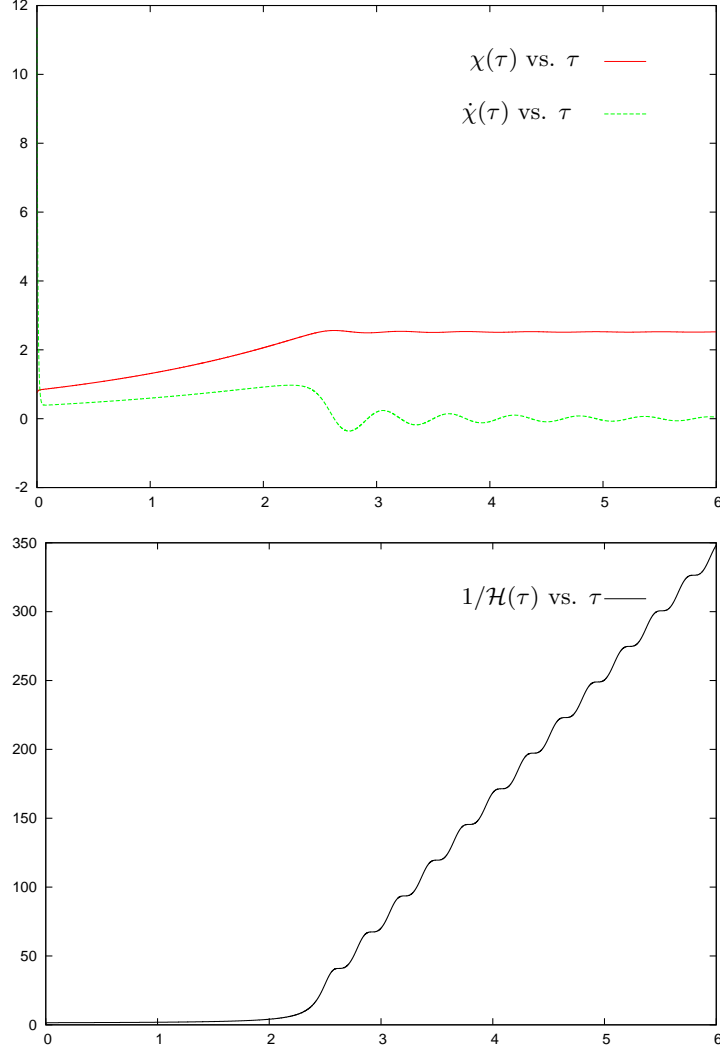


FIG. 8: Upper panel: $\chi(\tau)$ and $\dot{\chi}(\tau)$ as a function of the stretched cosmic time τ for $\chi(0) = 0.740$ and initial kinetic energy equal to the initial potential energy which implies $\dot{\chi}(0) = 12.6$. After a short fast-roll stage for $\tau \lesssim 0.0247$ the inflaton field slowly rolls toward its absolute minimum at $\chi = \sqrt{8/y} \simeq 2.52 \dots$, $\dot{\chi} = 0$. Lower panel: $1/\mathcal{H}$ vs. τ . $1/\mathcal{H}$ grows slowly during inflation $\tau < \tau_{end} \simeq 2.39$ and grows as $1/\mathcal{H} \simeq \frac{3}{2} N (\tau - \tau_{end})$ in the subsequent matter dominated era.

Since $M/M_{Pl} \sim 3 \times 10^{-3}$ [eq.(3.39)], we **naturally** find from eq.(3.31) the order of magnitude of the cubic and quartic couplings,

$$\lambda_3 \sim 10^{-6} m \quad , \quad \lambda = \lambda_4 \sim 10^{-12} \quad . \quad (3.41)$$

These relations are a **natural** consequence of the validity of the effective field theory and of slow-roll and solve the **fine tuning problem**. We emphasize that the ‘see-saw-like’ form of the couplings is a consequence of the form of the potential eq.(3.17) and is valid for all inflationary models within the class defined by eq.(3.17) [9].

We obtain from at the coupling value $y = 1.26$ that best fit the WMAP5+SDSS+SN data [5, 12]

$$M = 0.543 \times 10^{16} \text{ GeV} \quad , \quad m = 1.21 \times 10^{13} \text{ GeV} \quad \text{and} \quad H \simeq 6 \times 10^{13} \text{ GeV} \quad \text{for} \quad y = 1.26 \quad . \quad (3.42)$$

Notice that these values **agree** with the generic estimates eq.(3.39)-(3.40) which apply in order of magnitude to all inflationary potentials within the universal slow-roll class eq.(3.17).

The ratio r to leading order in $1/N$ is given by [5]

$$r = \frac{8}{N} \left[\frac{w'(\chi)}{w(\chi)} \right]^2 + \mathcal{O} \left(\frac{1}{N^2} \right) \quad . \quad (3.43)$$

The fact that $r \sim 1/N$ [eq.(3.43)] shows that the tensor fluctuations are suppressed by a factor $N \sim 60$ compared with the curvature scalar fluctuations. This suppression can be explained as follows: the scalar curvature fluctuations are quantum fluctuations around the classical inflaton while the tensor fluctuations are just quantum zero-point fluctuations.

The observation of a nonzero r will have **twofold** relevance. First, it would be the **first** detection of (linearized) gravitational waves as predicted by Einstein's General Relativity. Second, $r > 0$ indicates the presence of gravitons, namely, **quantized** gravitational waves at tree level.

Neutrino oscillations and neutrino masses m_ν are currently explained in the see-saw mechanism as follows [40],

$$\Delta m_\nu \sim \frac{M_{Fermi}^2}{M_R} \quad (3.44)$$

where $M_{Fermi} \sim 250$ GeV is the Fermi mass scale, $M_R \gg M_{Fermi}$ is a large energy scale and Δm_ν is the difference between the neutrino masses for different flavors. The observed values for $\Delta m_\nu \sim 0.009 - 0.05$ eV naturally call for a mass scale $M \sim 10^{15-16}$ GeV close to the GUT scale [40].

We see thus, that the energy scale $\sim 10^{16}$ GeV appears in fundamental physics in at least three independent ways: grand unification scale, inflation scale and the scale M_R in the see-saw neutrino mass formula.

C. Ginsburg-Landau polynomial realizations of the Inflaton Potential

In the Ginsburg-Landau spirit the potential is a polynomial in the field starting by a constant term [14]. Linear terms can always be eliminated by a constant shift of the inflaton field. The quadratic term can have a positive or a negative sign. In the first case the symmetry $\varphi \rightarrow -\varphi$ is unbroken (unless the potential contains terms odd in φ), in the latter case the symmetry $\varphi \rightarrow -\varphi$ is spontaneously broken since the minimum of the potential is at $\varphi \neq 0$.

Inflaton potentials with $w''(0) > 0$ lead to chaotic (large field) inflation while inflaton potentials with $w''(0) < 0$ lead to new (small field) inflation.

The inflaton potential must be bounded from below, therefore the next potential beyond the quadratic potential is the quartic one with a positive quartic coefficient.

The request of renormalizability restricts the degree of the inflaton potential to four. However, since the theory of inflation is an effective theory, potentials of degrees higher than four are acceptable.

In the context of the Ginsburg-Landau effective theory it is highly unnatural to set $m = 0$ [14]. This corresponds to be exactly at the critical point of the model where the mass vanishes, that is, in the statistical mechanical context the correlation length is infinite. In fact, the WMAP result convincingly excluding the $m = 0$ choice (purely φ^4 potential, see [5, 7, 8]) supports this purely theoretical argument against the φ^4 monomial potential. Therefore, from a physical point of view, the pure quartic potential is a weird choice implying to fine tune to zero the coefficient of the mass term.

Dropping the cubic term implies that $\varphi \rightarrow -\varphi$ is a symmetry of the inflaton potential. As stated in [10], we do not see reasons based on fundamental physics to choose a zero or a nonzero cubic term. However, the MCMC analysis of the WMAP plus LSS data shows that the cubic term can be ignored for new inflation (see [12]).

A model with only one field is clearly unrealistic since the inflaton would then describe a stable and ultra-heavy ($\sim 10^{13}$ GeV) particle. It is necessary to couple the inflaton with lighter particles, in which case the inflaton can decay into them. There are many available scenarios for inflation. Most of them add other fields coupled to the inflaton. This variety of inflationary scenarios may seem confusing since several of them are compatible with the observational data [7, 8]. Indeed, future observations should constraint the models more tightly excluding some families of them. The hybrid inflationary [38] models are amongst those strongly disfavoured by the WMAP data since they give $n_s > 1$ in most of their parameter space contrary to the WMAP results [8]. The regions of parameter space where hybrid inflation yields $n_s < 1$ are equivalently covered by one-field chaotic inflation [11].

The variety of inflationary models shows the **power** of the inflationary paradigm. Whatever the correct microscopic model for the early universe would be, it should include inflation with the generic features we know today. Many inflatons can be considered (multi-field inflation), but such family of models introduce extra features as non-adiabatic (isocurvature) density fluctuations, which in turn become strongly constrained by the WMAP data [7, 8].

IV. OVERVIEW OF DARK MATTER

A **model independent** analysis of dark matter (DM) both decoupling ultrarelativistic (UR) and non-relativistic (NR) based on the DM phase-space density $\mathcal{D} = \rho_{DM}/\sigma_{DM}^3$ is presented in refs. [42, 43, 69]. We derived in [43] explicit

formulas for the DM particle mass m and for the number of ultrarelativistic degrees of freedom g_d at decoupling (and equivalently for the decoupling temperature T_d). We find that for DM particles decoupling UR both at local thermal equilibrium (LTE) and out of LTE, m turns to be at the **keV scale**. For example, for DM Majorana fermions decoupling at LTE the mass results $m \simeq 0.85$ keV. For DM particles decoupling NR, $\sqrt{m T_d}$ results in the **keV scale** (T_d is the decoupling temperature) and the m value is consistent with the keV scale. Also, lower and upper bounds on the DM annihilation cross-section for NR decoupling are derived. We evaluate the free-streaming (Jeans') wavelength and Jeans' mass: they result independent of the type of DM except for the DM self-gravity dynamics. The free-streaming wavelength today results in the **kpc range**. These results are based on our theoretical analysis, astronomical observations of dwarf spheroidal satellite galaxies in the Milky Way and N -body numerical simulations. We analyze and discuss the results on \mathcal{D} from analytic approximate formulas both for linear fluctuations and the (non-linear) spherical model and from N -body simulations results. We obtain in this way upper bounds for the DM particle mass which all result below the 100 keV range [43].

A. The dark matter particle mass

Dark matter constitutes 83 % of the matter in the Universe. Its nature is still unknown. The external part of galaxies (halo) is formed by dark matter. Baryonic matter dominates the internal parts of luminous galaxies.

Dark matter (DM) must be non-relativistic by the time of structure formation ($z < 30$) in order to reproduce the observed small structure at $\sim 2 - 3$ kpc.

DM particles can decouple being ultrarelativistic (UR) at $T_d \gg m$ or being non-relativistic (NR) at $T_d \ll m$ where m is the mass of the DM particles and T_d the decoupling temperature. We consider DM particles that decouple **at** or **out** of local thermal equilibrium (LTE) [42, 43].

The DM distribution function F_d freezes out at decoupling. Therefore, for all times after decoupling F_d coincides with its expression at decoupling. F_d is a function of T_d , m and the comoving momentum of the DM particles p_c .

Knowing the distribution function $F_d(p_c)$, we can compute physical magnitudes as the DM velocity fluctuations and the DM energy density. For the relevant times t during structure formation, when the DM particles are non-relativistic, we have

$$\langle \vec{V}^2 \rangle(t) = \langle \frac{\vec{p}_{ph}^2}{m^2} \rangle(t) = \frac{\int \frac{d^3 p_{ph}}{(2\pi)^3} \frac{\vec{p}_{ph}^2}{m^2} F_d[a(t) p_{ph}]}{\int \frac{d^3 p_{ph}}{(2\pi)^3} F_d[a(t) p_{ph}]} \quad (4.1)$$

where we use the physical momentum of the DM particles $p_{ph}(t) \equiv p_c/a(t)$ as integration variable. The physical momentum $p_{ph}(t)$ coincides today with the comoving momentum p_c .

We can relate the covariant decoupling temperature T_d , the effective number of UR degrees of freedom at decoupling g_d and the photon temperature today T_γ by using entropy conservation [2, 32, 46]:

$$T_d = \left(\frac{2}{g_d} \right)^{\frac{1}{3}} T_\gamma, \quad \text{where} \quad T_\gamma = 0.2348 \text{ meV} \quad \text{and} \quad 1 \text{ meV} = 10^{-3} \text{ eV}. \quad (4.2)$$

The DM energy density can be written as

$$\rho_{DM}(t) = g \int \frac{d^3 p_{ph}}{(2\pi)^3} \sqrt{m^2 + p_{ph}^2} F_d[a(t) p_{ph}], \quad (4.3)$$

where g is the number of internal degrees of freedom of the DM particle, typically $1 \leq g \leq 4$.

By the time when the DM particles are non-relativistic, the energy density eq.(4.3) becomes

$$\rho_{DM}(t) = \frac{m}{2} \frac{g}{\pi^2} \frac{T_d^3}{a^3(t)} I_2 \equiv m n(t), \quad (4.4)$$

where

$$I_2 \equiv \int_0^\infty y^2 F_d(y) dy,$$

$n(t)$ is the number of DM particles per unit volume and we used as integration variable

$$y \equiv \frac{p_{ph}(t)}{T_d(t)} = \frac{p_c}{T_d}. \quad (4.5)$$

From eq.(4.4) at $t = 0$ and from the value observed today for ρ_{DM} [table I and eq.(2.7)], we find the value of the DM mass:

$$m = \pi^2 \Omega_{DM} \frac{\rho_c}{T_\gamma^3} \frac{g_d}{g} \frac{I_4}{I_2} = 6.986 \text{ eV} \frac{g_d}{g} \frac{I_4}{I_2}, \quad (4.6)$$

where ρ_c is the critical density.

Using as integration variable y [eq.(4.5)], eq.(4.1) for the velocity fluctuations, yields

$$\langle \vec{V}^2 \rangle(t) = \left[\frac{T_d}{m a(t)} \right]^2 \frac{I_4}{I_2}, \quad (4.7)$$

where

$$I_4 \equiv \int_0^\infty y^4 F_d(y) dy.$$

Expressing T_d in terms of the CMB temperature today according to eq.(4.2) gives for the one-dimensional velocity dispersion,

$$\sigma_{DM}(z) = \sqrt{\frac{1}{3} \langle \vec{V}^2 \rangle(z)} = \frac{2^{\frac{1}{3}}}{\sqrt{3}} \frac{1+z}{g_d^{\frac{1}{3}}} \frac{T_\gamma}{m} \sqrt{\frac{I_4}{I_2}} = 0.05124 \frac{1+z}{g_d^{\frac{1}{3}}} \frac{\text{keV}}{m} \left[\frac{I_4}{I_2} \right]^{\frac{1}{2}} \frac{\text{km}}{\text{s}}. \quad (4.8)$$

It is very useful to consider the phase-space density invariant under the universe expansion [42, 47]

$$\mathcal{D}(t) \equiv \frac{n(t)}{\langle \vec{P}_{ph}^2(t) \rangle^{\frac{3}{2}}} \stackrel{\text{non-rel}}{=} \frac{1}{3 \sqrt{3}} \frac{1}{m^4} \frac{\rho_{DM}(t)}{\sigma_{DM}^3(t)}, \quad (4.9)$$

where we consider the relevant times t during structure formation when the DM particles are non-relativistic. $\mathcal{D}(t)$ is a *constant* in absence of self-gravity. In the non-relativistic regime $\mathcal{D}(t)$ can only **decrease** by collisionless phase mixing or self-gravity dynamics [48].

We derive a useful expression for the phase-space density \mathcal{D} from eqs.(4.4), (4.8) and (4.9) with the result

$$\mathcal{D} = \frac{g}{2 \pi^2} \frac{I_2^{\frac{5}{2}}}{I_4^{\frac{3}{2}}}, \quad (4.10)$$

Observing dwarf spheroidal satellite galaxies in the Milky Way (dSphs) yields for the phase-space density today [49]:

$$\frac{\rho_s}{\sigma_s^3} \sim 5 \times 10^3 \frac{\text{keV}/\text{cm}^3}{(\text{km/s})^3} = (0.18 \text{ keV})^4. \quad (4.11)$$

The precision of these results is about a factor 10.

After the radiation dominated era the phase-space density reduces by a factor that we call Z

$$\mathcal{D}(0) = \frac{1}{Z} \mathcal{D}(z \sim 3200) \quad (4.12)$$

Recall that $\mathcal{D}(z)$ [eq.(4.9)] is independent of z for $z \gtrsim 3200$ since density fluctuations were $\lesssim 10^{-3}$ before the matter dominated era [3].

The range of values of Z (which is necessarily $Z > 1$) is analyzed in detail in ref. [43].

We can express the phase-space density today from eqs.(4.9) and (4.11) as

$$\mathcal{D}(0) = \frac{1}{3 \sqrt{3}} \frac{1}{m^4} \frac{\rho_s}{\sigma_s^3}. \quad (4.13)$$

Therefore, eqs.(4.9), (4.12) and (4.13) yield,

$$\frac{\rho_s}{\sigma_s^3} = \frac{1}{Z} \frac{\rho_{DM}}{\sigma_{DM}^3}(z \sim 3200), \quad (4.14)$$

where $\rho_{DM}/\sigma_{DM}^3(z \sim 3200)$ follows from eqs.(4.9) and (4.10),

$$\frac{\rho_{DM}}{\sigma_{DM}^3}(z \sim 3200) = \frac{3 \sqrt{3} m^4}{2 \pi^2} g \frac{I_2^{\frac{5}{2}}}{I_4^{\frac{3}{2}}}. \quad (4.15)$$

We can express m from eqs.(4.11)-(4.15) in terms of \mathcal{D} and observable quantities as

$$m^4 = \frac{Z}{3 \sqrt{3}} \frac{\rho_s}{\mathcal{D} \sigma_s^3} = \frac{2 \pi^2}{3 \sqrt{3}} \frac{Z}{g} \frac{\rho_s}{\sigma_s^3} \frac{I_4^{\frac{3}{2}}}{I_2^{\frac{5}{2}}}, \quad (4.16)$$

$$m = 0.2504 \left(\frac{Z}{g} \right)^{\frac{1}{4}} \frac{I_4^{\frac{3}{8}}}{I_2^{\frac{5}{8}}} \text{ keV}. \quad (4.17)$$

Combining this with eq.(4.6) for m we obtain the number of ultrarelativistic degrees of freedom at decoupling as

$$g_d = \frac{2^{\frac{1}{4}}}{3^{\frac{3}{8}} \pi^{\frac{3}{2}}} \frac{g^{\frac{3}{4}}}{\Omega_{DM}} \frac{T_\gamma^3}{\rho_c} \left(\frac{Z \rho_s}{\sigma_s^3} \right)^{\frac{1}{4}} [I_2 I_4]^{\frac{3}{8}} = 35.96 Z^{\frac{1}{4}} g^{\frac{3}{4}} [I_2 I_4]^{\frac{3}{8}}. \quad (4.18)$$

A succession of several violent phases happens during the structure formation stage ($z \lesssim 30$). Their cumulated effect together with the evolution of \mathcal{D} for $3200 \gtrsim z \gtrsim 30$ produces a range of values of the Z factor which we can conservatively estimate on the basis of the N -body simulations results [50] and the approximation results from the linear approximation and the spherical model [43]. This gives a range of values $1 < Z < 10000$ for dSphs [43].

B. Jeans' (free-streaming) wavelength and Jeans' mass

It is very important to evaluate the Jeans' length and Jeans' mass in the present context [46, 51, 52]. The Jeans' length is analogous to the free-streaming wavelength. The free-streaming wavevector is the largest wavevector exhibiting gravitational instability and characterizes the scale of suppression of the DM transfer function during matter domination [53].

The physical free-streaming wavelength can be expressed as [46, 53]

$$\lambda_{fs}(t) = \lambda_J(t) = \frac{2 \pi}{k_{fs}(t)} \quad (4.19)$$

where $k_{fs}(t) = k_J(t)$ is the physical free-streaming wavenumber given by

$$k_{fs}^2(t) = \frac{4 \pi G \rho_{DM}(t)}{\langle \vec{V}^2 \rangle(t)} = \frac{3}{2} [1 + z(t)] \frac{H_0^2 \Omega_{DM}}{\langle \vec{V}^2 \rangle(0)}. \quad (4.20)$$

where we used that $\rho_{DM}(t) = \rho_{DM}(0) (1 + z)^3$, table I and eq.(2.7).

We obtain the primordial DM dispersion velocity σ_{DM} from eqs. (4.4), (2.7) and (4.14),

$$\sqrt{\frac{1}{3} \langle \vec{V}^2 \rangle(0)} = \sigma_{DM} = \left(3 M_{Pl}^2 H_0^2 \Omega_{DM} \frac{1}{Z} \frac{\sigma_s^3}{\rho_s} \right)^{\frac{1}{3}} \quad (4.21)$$

This expression is valid for **any kind** of DM particles. Inserting eq.(4.21) into eq.(4.20) yields for the physical free-streaming wavelength

$$\lambda_{fs}(z) = \frac{2 \sqrt{2} \pi}{\Omega_{DM}^{\frac{1}{6}}} \left(\frac{3 M_{Pl}^2}{H_0} \right)^{\frac{1}{3}} \left(\frac{\sigma_s^3}{Z \rho_s} \right)^{\frac{1}{3}} \frac{1}{\sqrt{1+z}} = \frac{16.3}{Z^{\frac{1}{3}}} \frac{1}{\sqrt{1+z}} \text{ kpc}. \quad (4.22)$$

where we used $1 \text{ keV} = 1.563738 \cdot 10^{29} (\text{kpc})^{-1}$.

Notice that λ_{fs} and therefore λ_J turn to be **independent** of the nature of the DM particle except for the factor Z .

As stated above Z for dSphs is in the range:

$$1 < Z < 10000.$$

Therefore, $1 < Z^{\frac{1}{3}} < 21.5$ and the free-streaming wavelength results in the range

$$0.757 \frac{1}{\sqrt{1+z}} \text{ kpc} < \lambda_{fs}(z) < 16.3 \frac{1}{\sqrt{1+z}} \text{ kpc} .$$

These values at $z = 0$ are consistent with the N -body simulations reported in [54] and are of the order of the small DM structures observed today [49].

The Jeans' mass is given by

$$M_J(t) = \frac{4}{3} \pi \lambda_J^3(t) \rho_{DM}(t) . \quad (4.23)$$

and provides the smallest unstable mass by gravitational collapse [2, 46]. Inserting here eq.(4.4) for the DM density and eq.(4.22) for $\lambda_J(t) = \lambda_{fs}(t)$ yields

$$M_J(z) = 192 \sqrt{2} \pi^4 \sqrt{\Omega_{DM}} M_{Pl}^4 H_0 \frac{\sigma_s^3}{Z \rho_s} (1+z)^{\frac{3}{2}} = \frac{0.4464}{Z} 10^7 M_{\odot} (1+z)^{\frac{3}{2}} . \quad (4.24)$$

Taking into account the Z -values range yields

$$0.4464 10^3 M_{\odot} < M_J(z) (1+z)^{-\frac{3}{2}} < 0.4464 10^7 M_{\odot} .$$

This gives masses of the order of galactic masses $\sim 10^{11} M_{\odot}$ by the beginning of the MD era $z \sim 3200$. In addition, the comoving free-streaming wavelength scale by $z \sim 3200$

$$3200 \times \lambda_{fs}(z \sim 3200) \sim 100 \text{ kpc} ,$$

turns to be of the order of the galaxy sizes today.

C. Dark Matter Decoupling at Local Thermal Equilibrium (LTE)

If the dark matter particles of mass m decoupled at a temperature $T_d \gg m$ their freezed-out distribution function only depends on

$$\frac{p_c}{T_d} = \frac{p_{ph}(t)}{T_d(t)}, \quad \text{where} \quad T_d(t) \equiv \frac{T_d}{a(t)} .$$

That is, the distribution function for dark matter particles that decoupled in thermal equilibrium takes the form

$$F_d^{equil} \left[\frac{p_{ph}(t)}{T_d(t)} \right] = F_d^{equil} \left[\frac{p_c}{T_d} \right] ,$$

where F_d^{equil} is a Bose-Einstein or Fermi-Dirac distribution function:

$$F_d^{equil}[p_c] = \frac{1}{\exp[\sqrt{m^2 + p_c^2}/T_d] \pm 1} . \quad (4.25)$$

Notice that for eq.(4.25) in this regime:

$$\frac{\sqrt{m^2 + p_c^2}}{T_d} \stackrel{T_d \gg m}{\approx} y + \mathcal{O} \left(\frac{m^2}{T_d^2} \right) .$$

where y is defined by eq.(4.5) and we can use as distribution functions

$$F_d^{equil}(y) = \frac{1}{e^y \pm 1} . \quad (4.26)$$

Using eqs.(3.20) and (4.25), we find then for Fermions and for Bosons decoupling at LTE

$$m = \frac{g_d}{g} \begin{cases} 3.874 \text{ eV} & \text{Fermions} \\ 2.906 \text{ eV} & \text{Bosons} \end{cases} . \quad (4.27)$$

Approximation used	Upper limit on Z	Upper limit on $m \simeq 0.5 Z^{\frac{1}{4}}$ keV
Linear fluctuations	$\sim 1.3 \times 10^{11}$	96 keV
Spherical Model	$\sim 1.29 \times \delta_i^{-\frac{3}{2}} \simeq 4.1 \times 10^4$	7.1 keV

TABLE II: Upper bounds for the Z -factor [defined by eq.(4.12)] and for the mass of the DM particle obtained for two different approximation methods. Notice that only the spherical model takes into account non-linear self-gravity effects. The mass m **mildly** depends on Z through the power $1/4$. In any case m results in the keV range.

We see that for DM that decoupled at the Fermi scale: $T_d \sim 100$ GeV and $g_d \sim 100$, m results in the keV scale as already remarked in ref. [52, 65]. DM particles may decouple earlier with $T_d > 100$ GeV but g_d is always in the hundreds even in grand unified theories where T_d can reach the GUT energy scale. Therefore, eq.(4.27) **strongly suggests** that the mass of the DM particles which decoupled UR in LTE is in the **keV scale**.

It should be noticed that the Lee-Weinberg [67] lower bound as well as the Cowsik-McClelland [66] upper bound follow from eq.(4.6) as shown in ref.[42].

Computing the integrals in eq.(4.10) with the distribution functions eq.(4.25) yields for DM decoupling UR in LTE

$$\mathcal{D} = g \begin{cases} \frac{1}{4\pi^2} \sqrt{\frac{\zeta^5(3)}{15\zeta^3(5)}} = 1.9625 \times 10^{-3} & \text{Fermions} \\ \frac{1}{8\pi^2} \sqrt{\frac{\zeta^5(3)}{3\zeta^3(5)}} = 3.6569 \times 10^{-3} & \text{Bosons} \end{cases} \quad (4.28)$$

where $\zeta(3) = 1.2020569 \dots$ and $\zeta(5) = 1.0369278 \dots$

Inserting the distribution function eq.(4.26) into eqs.(4.16) and (4.18) for m and g_d , respectively, we obtain

$$m = \left(\frac{Z}{g}\right)^{\frac{1}{4}} \text{ keV} \begin{cases} 0.568 & \text{Fermions} \\ 0.484 & \text{Bosons} \end{cases}, \quad g_d = g^{\frac{3}{4}} Z^{\frac{1}{4}} \begin{cases} 155 & \text{Fermions} \\ 180 & \text{Bosons} \end{cases}. \quad (4.29)$$

Since $g = 1 - 4$, for DM particle decoupling at LTE, we see from eq.(4.29) that $g_d > 100$ and thus, the DM particle should decouple for $T_d > 100$ GeV. Notice that $1 < Z^{\frac{1}{4}} < 10$ for $1 < Z < 10000$.

We can express the free-streaming wavelength as a function of the DM particle mass from eqs.(4.22) and (4.29) with the result,

$$\lambda_{fs}(z) = \left(\frac{\text{keV}}{m}\right)^{\frac{4}{3}} \frac{\text{kpc}}{g^{\frac{1}{3}}} \frac{1}{\sqrt{1+z}} \begin{cases} 7.67 & \text{Fermions} \\ 6.19 & \text{Bosons} \end{cases}. \quad (4.30)$$

D. The DM particle at the keV scale: conclusions

Our results on DM are **independent** of the particle model that will describe the dark matter. We consider both DM particles that decouple being NR and UR and both decoupling at LTE and out of LTE [42, 43, 69]. Our analysis and results refer to the mass of the dark matter particle and the number of ultrarelativistic effective degrees of freedom when the DM particles decoupled. We do not make assumptions about the nature of the DM particle and we only assume that its non-gravitational interactions can be neglected in the present context (which is consistent with structure formation and observations).

For DM particles decoupling ultrarelativistic and out of thermal equilibrium the results for m and g_d on the same scales as decoupling at LTE [43].

When DM particles decouple being non-relativistic ($T_d < m$) the analysis is slightly more elaborated since it also involves the total annihilation cross-section. We obtain for non-relativistic DM particles decoupling at LTE [43],

$$\sqrt{m T_d} = 1.47 \left(\frac{Z}{g_d}\right)^{\frac{1}{3}} \text{ keV}. \quad (4.31)$$

Therefore, the combination $\sqrt{m T_d}$ must be in the keV scale for the NR decoupling case.

In case DM particles explain the formation of galactic center black holes, DM particles must be fermions with keV-scale mass [68].

The mass for the DM particle in the keV range is much larger than the temperature during the MD era, hence dark matter is **cold** (CDM).

A possible CDM candidate in the keV scale is a sterile neutrino [55] produced via their mixing and oscillation with an active neutrino species. Other putative CDM candidates in the keV scale are the gravitino [56], the light neutralino [57] and the majoron [58].

Actually, many more extensions of the Standard Model of Particle Physics can be envisaged to include a DM particle with mass in the keV scale and weakly enough coupled to the Standard Model particles.

Lyman- α forest observations provide indirect lower bounds on the masses of sterile neutrinos [59] while constraints from the diffuse X-ray background yield upper bounds on the mass of a putative sterile neutrino DM particle [60]. All these recent constraints are consistent with DM particle masses at the keV scale.

The DAMA/LIBRA collaboration has confirmed the presence of a signal in the keV range [61]. Whether this signal is due to DM particles in the keV mass scale is still unclear [62]. On the other hand, the DAMA/LIBRA signals cannot be explained by a hypothetical WIMP particle with mass $\gtrsim O(1)$ GeV since this would be in conflict with previous WIMPS direct detection experiments [63].

We find for typical wimps with $m = 100$ GeV, $T_d = 5$ GeV [32] and therefore $g_d \simeq 80$ [2]. This requires from eq.(4.31) a huge $Z \sim 10^{23}$, well above the upper bounds displayed in Table II [43]. Hence, wimps cannot reproduce the observed galaxy properties. In addition, recall that $Z \sim 10^{23}$ produces from eq.(4.22) an extremely short λ_{fs} today

$$\lambda_{fs}(0) \sim 3.51 \cdot 10^{-4} \text{ pc} = 72.4 \text{ AU} .$$

No galactic structures has been observed at such solar system scales.

Further evidence for the DM particle mass in the keV scale follows by contrasting the observed value of the constant surface density of galaxies to the theoretical calculation from the linearized Boltzmann-Vlasov equation [69].

In summary, our analysis shows that DM particles decoupling UR in LTE have a mass m in the keV scale with $g_d \gtrsim 150$ as shown in sec. IV C. That is, decoupling happens at least at the 100 GeV scale. The values of m and g_d may be smaller for DM decoupling UR out of LTE than for decoupling UR in LTE [43]. For DM particles decoupling NR in LTE we find that $\sqrt{m T_d}$ is in the keV range (see eq.(4.31) and [43]). This is consistent with the DM particle mass in the keV range.

Notice that the present uncertainty by one order of magnitude of the observed values of the phase-space density ρ_s/σ_s^3 only affects the DM particle mass through a power 1/4 of this uncertainty according to eqs. (4.16)-(4.17). Namely, by a factor $10^{\pm 1} \simeq 1.8$.

We find that the free streaming wavelength (Jeans' length) is **independent** of the nature of the DM particle except for the Z factor characterizing the decrease of the phase-space density through self-gravity [sec. IV B]. The values found for the Jeans' length and the Jeans' mass for m in the keV scale are consistent with the observed small structure and with the masses of the galaxies, respectively.

Independent further evidence for the DM particle mass in the keV scale were recently given in [64]. (See also [49]). DM particles with mass in the keV scale can alleviate CDM problems as the satellite problem [70] and the voids problem [71]. The DM particle mass in the keV explain why DM particles were not found in detectors sensitive to particles heavier than ~ 1 GeV [72]. In addition, astrophysical mechanisms that can explain the e^+ and \bar{p} excess in cosmic rays without requiring DM particles in the GeV scale or above were put forward in [73].

V. OUTLOOK AND FUTURE PERSPECTIVES

This short review presents the state of the art of the effective theory of inflation and its successful confrontation with the CMB and LSS data. We can highlight as perspectives for a foreseeable future:

- Measurement of the tensor/scalar ratio r by CMB experiments as Planck and the future satellite CMBPol. This would be the **first** detection of (linearized) gravitational waves as predicted by Einstein's General Relativity. In addition, since such primordial gravitational waves were born as quantum fluctuations, this would be the **first** detection of gravitons, namely, **quantized** gravitational waves at tree level. Such detection of the primordial gravitational waves will test our prediction $r \simeq 0.05$ based on the effective theory of slow-roll inflation (broken symmetric binomial and trinomial potentials) [12, 21].
- The running of the spectral index $dn_s/d \ln k$. Since the range of the cosmologically relevant modes is $\Delta \ln k < 9$, we have $\Delta n_s < 9/N^2 \sim 0.0025$, where we use the leading order in slow-roll [5]. Therefore, the effective theory of slow-roll inflation indicates that the detection of the running calls for measurements of n_s with a one per thousand precision on a wide range of wavenumbers.

- Non-gaussianity measurements. Although this subject is beyond the scope of this short review, let us recall that primordial non-gaussianity is of the order $f_{NL} \sim 1/N$ in single-field slow-roll inflation [41]. Such small primordial non-gaussianity is hardly expected to be measured in a foreseeable future.
- More precise measurements of n_s together with better data on r and $dn_s/d\ln k$ will permit to better select the correct inflationary model. This will test our prediction that a broken symmetric inflaton potential with moderate nonlinearity (new inflation) best describes the data [5, 12, 21].
- Direct dark matter particle detection. Unfortunately, all experiments in course or planned are built to detect dark matter particles in the GeV scale or heavier. The same applies for eventual wimps production at the LHC. A keV scale DM particle will certainly not be detected in such experiments.
- Astrophysical dark matter. Theoretical work, more abundant and better data from galaxies and N -body computer simulations with keV-scale DM will certainly provide relevant new answers to the current problems in N -body computer simulations using heavy ($m > 1$ GeV) DM particles. By current problems we mean the satellite problem, the core-cusp problem, the void problem and may be the angular momentum problem. Precise values for m and g_d should be obtained.

-
- [1] Kazanas D, *ApJ* **241**, L59 (1980); Guth A, *Phys. Rev.* **D23**, 347 (1981); Sato K, *MNRAS*, **195**, 467 (1981).
- [2] Kolb EW and Turner MS, *The Early Universe*, Addison Wesley. Redwood City, C.A. 1990.
- [3] Dodelson S, *Modern Cosmology*, Academic Press, 2003.
- [4] See for example: Hu W., Dodelson S., *Ann. Rev. Astron. Ap.* 40: 171 (2002); Lidsey J, Liddle A, Kolb E, Copeland E, Barreiro T, Abney M, *Rev. of Mod. Phys.* 69: 373, (1997). Hu W., astro-ph/0402060. Mukhanov VF, Feldman HA, Brandenberger RH, *Phys. Rep.* 215:203 (1992). A. Riotto, hep-ph/0210162. Liddle AR, Lyth DH, *Cosmological Inflation and Large Scale Structure*, Cambridge University Press, 1999.
- [5] D. Boyanovsky, C. Destri, H. J. de Vega, N. G. Sánchez, arXiv:0901.0549, International Journal of Modern Physics **A 24**, 3669-3864 (2009).
- [6] Cole et al. *MNRAS*, 362, 505 (2005). Eisenstein et al. *ApJ*, 633, 560 (2005). W. J. Percival et al. *Ap. J.* 657 (2007) 51 and *MNRAS*, 381 (2007) 1053.
- [7] C. L. Bennett *et.al.* (WMAP collaboration), *Ap. J. Suppl.* **148**, 1 (2003).
A. Kogut *et.al.* (WMAP collaboration), *Ap. J. Suppl.* **148**, 161 (2003).
D. N. Spergel *et. al.* (WMAP collaboration), *Ap. J. Suppl.* **148**, 175 (2003).
H. V. Peiris *et.al.* (WMAP collaboration), *Ap. J. Suppl.* **148**, 213 (2003).
- [8] E. Komatsu et al.(WMAP collaboration), *Astrophys. J. Suppl.* 180:330 (2009).
G. Hinshaw et al.(WMAP collaboration), *Astrophys. J. Suppl.* 180:225 (2009).
M. R. Nolta et al.(WMAP collaboration), *Astrophys. J. Suppl.* 180:296 (2009).
E. Komatsu et al.(WMAP collaboration), arXiv:1001.4538.
D. Larson et al.(WMAP collaboration), arXiv:1001.4635.
N. Jarosik et al.(WMAP collaboration), arXiv:1001.4744.
- [9] D. Boyanovsky, H. J. de Vega, N. G. Sánchez, *Phys. Rev.* **D 73**, 023008 (2006).
- [10] D. Cirigliano, H. J. de Vega, N. G. Sánchez, *Phys. Rev.* **D 71**, 103518 (2005).
- [11] H. J. de Vega, N. G. Sánchez, *Phys. Rev.* **D 74**, 063519 (2006).
- [12] C. Destri, H. J. de Vega, N. G. Sánchez, astro-ph/0703417, *Phys. Rev.* **D77**, 043509 (2008).
- [13] G. F. Smoot *et. al.* (COBE collaboration), *Astro. Phys. Jour.* **396**, 1 (1992).
- [14] Statistical Physics, vol 9, E M Lifshitz, L P Pitaevsky, Pergamon Press, Oxford 1980, see secs. 142 part I and 45 part II.
L. D. Landau, *Zh. Eksp. Teor. Fiz.*, **7**, 19 (1937) and **7**, 545 (1937) and in *Collected Papers of L. D. Landau*, Pergamon Press, Oxford, 1965. V. L. Ginsburg, *Zh. Eksp. Teor. Fiz.* **15**, 739 and **10**, 107 (1945). V. L. Ginsburg, L. D. Landau, *Zh. Eksp. Teor. Fiz.* **20**, 1064 (1950). V. L. Ginsburg, *About Science, Myself and Others*, Part I, Chapters 5-7, IoP, Bristol, 2005.
- [15] H. Leutwyler, *Ann. Phys.* 235, 165 (1994), hep-ph/9409423. S. Weinberg, hep-ph/9412326 and ‘The Quantum Theory of Fields’, vol. 2, Cambridge University Press, Cambridge, 2000.
- [16] D. Boyanovsky, H. J. de Vega, C. M. Ho, N. G. Sánchez, *Phys. Rev.* **D75**, 123504 (2007).
- [17] D. Boyanovsky, H. J. de Vega, N. G. Sánchez, *Nucl. Phys.* **B747**, 25 (2006).
- [18] D. Boyanovsky, H. J. de Vega, N. G. Sánchez, *Phys. Rev.* **D 72**, 103006 (2005).
- [19] D. Boyanovsky, H. J. de Vega, N. G. Sánchez, *Phys. Rev.* **D 74**, 123006 (2006).
- [20] D. Boyanovsky, H. J. de Vega, N. G. Sánchez, *Phys. Rev.* **D 74**, 123007 (2006).
- [21] C. Destri, H. J. de Vega, N. G. Sánchez, *Phys. Rev.* **D 78**, 023013 (2008).
- [22] M. Kowalski *et al.*, *Astrophys. J.* 686, 749 (2008).

- [23] D. Boyanovsky, H. J. de Vega, in *Astrofundamental Physics*, NATO ASI series vol. 562, 2000, Lectures at the Chalonge School, astro-ph/0006446. D. Boyanovsky, F. J. Cao, H. J. de Vega, *Nucl. Phys. B* **632**, 121 (2002). D. Boyanovsky, D. Cormier, H. J. de Vega, R. Holman, *Phys. Rev. D* **55**: 3373 (1997); D. Boyanovsky, D. Cormier, H. J. de Vega, R. Holman, A. Singh, M. Srednicki, *Phys. Rev. D* **56**: 1939 (1997). D. Boyanovsky, H. J. de Vega, R. Holman, *Phys. Rev. D* **49**, 2769 (1994). D. Boyanovsky, H. J. de Vega, R. Holman, J. F. J. Salgado, *Phys. Rev. D* **54** 7570, (1996).
- [24] J. Traschen, R. Brandenberger, *Phys. Rev. D* **42**, 2491 (1990), Y. Shtanov, J. Traschen, R. Brandenberger, *Phys. Rev. D* **51**, 5438 (1995), L. Kofman, A. Linde, A. Starobinsky, *Phys. Rev. Lett.* **73**, 3195 (1994); *Phys. Rev. Lett.* **76**, 1011 (1996). A. Linde, in *Current Topics in Astrofundamental Physics, 'The Early Universe'*, Proceedings of the Chalonge Erice School, N. Sánchez and A. Zichichi Editors, Nato ASI series C, vol. 467, 1995, Kluwer Acad. Publ.
- [25] D. I. Kaiser, *Phys. Rev. D* **53** (1996) 1776.
- [26] J. Baacke, C. Patzold, *Phys. Rev. D* **62**, 084008 (2000). ; *ibid* **D61**:024016,2000. J. Baacke, K. Heitmann, C. Patzold, *Phys. Rev. D* **58**:125013,1998; *ibid* **D57**:6406,1998 and **D56**:6556,1997.
- [27] S. A. Ramsey, B. L. Hu, *Phys. Rev. D* **56**. 661, (1997); *ibid* 678, (1997). S. A. Ramsey, B. L. Hu, A. M. Stylianopoulos, *Phys. Rev. D* **57**:6003,1998.
- [28] D. Boyanovsky, D. Cormier, H. J. de Vega, R. Holman, S. Prem Kumar, *Phys. Rev. D* **57**, 2166 (1998).
- [29] D. Boyanovsky, F. J. Cao, H. J. de Vega, *Nucl. Phys. B* **632**: 121 (2002).
- [30] F. J. Cao, H. J. de Vega, N. Sanchez, *Phys. Rev. D* **70**: 083528 (2004).
- [31] D. Boyanovsky, C. Destri, H. J. de Vega, *Phys. Rev. D* **69**, 045003 (2004).
- [32] C. Destri, H. J. de Vega, *Phys. Rev. D* **73**, 025014 (2006).
- [33] A. Albrecht, P J Steinhardt, *Phys Rev Lett* **48**, 1220 (1982). A D Linde, *Phys Lett* **108B**, 389 (1982).
- [34] A D Linde, *Phys Lett* **129B**, 177 (1983).
- [35] W.-M. Yao et al., *Journal of Physics G* **33**, 1 (2006).
- [36] D. Boyanovsky, hep-ph/0102120, in the Proceedings of the Nato ASI, Eds, H. J. de Vega, I. Khalatnikov, N. Sanchez, p. 3, Kluwer, Dordrecht, 2001.
- [37] D. Boyanovsky, H. J. de Vega, D. J. Schwarz, hep-ph/0602002, *Ann. Rev. Nucl. Part. Sci.* **56**, 441-500, (2006).
- [38] V. A. Belinsky, L. P. Grishchuk, Ya. B. Zeldovich, I. M. Khalatnikov, *Phys. Lett. B* **155**, 232, (1985), *JETP* **62**, 195 (1985).
- [39] D. Boyanovsky, H. J. de Vega, *Phys. Rev. D* **70**, 063508 (2004).
- [40] D. Boyanovsky, H. J. de Vega, N. G. Sánchez, *Phys. Rev. D* **71**, 023509 (2005).
- [41] A. Lewis, S. Bridle, *Phys. Rev. D* **66**, 103511 (2002). <http://cosmologist.info/cosmomc/>
- [42] C. L. Reichardt *et al.*, *Astrophys. J.* **694**, 1200 (2009).
- [43] A. Linde, *Phys. Rev. D* **49**, 748 (1994). J. García Bellido, A. Linde, *Phys. Rev. D* **57**, 6075 (1998). E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart, J. Wands, *Phys. Rev. D* **49**, 6410 (1994).
- [44] F. C. Adams, J. R. Bond, K. Freese, J. A. Frieman, A. V. Olinto, *Phys. Rev. D* **47**, 426 (1993).
- [45] See for example, R. D. Peccei, *Nucl. Phys. Proc. Suppl.* **137** (2004) 277.
- [46] G. Altarelli, F. Feruglio, *New J. Phys.* **6** (2004) 106.
- [47] N. Bartolo, E. Komatsu, S. Matarrese, A. Riotto, *Phys. Rept.* **402**, 103-266 (2004) and references therein.
- [48] V. Acquaviva, N. Bartolo, S. Matarrese, A. Riotto, *Nucl. Phys. B* **667**, 119 (2003).
- [49] D. Boyanovsky, H. J. de Vega, N. Sanchez, *Phys. Rev. D* **77**, 043518 (2008)
- [50] H. J. de Vega, N. Sanchez, arXiv:0901.0922, *Mon. Not. R. Astron. Soc.* **404**, 885 (2010).
- [51] C. Destri, H. J. de Vega, N. G. Sanchez, arXiv:0906.4102.
- [52] C. Destri, H. J. de Vega, N. G. Sanchez, *Phys. Rev. D* **81**, 063520 (2010).
- [53] G. Börner, *The Early Universe*, Springer, 2003.
- [54] C. J. Hogan, J. J. Dalcanton, *Phys. Rev. D* **62**, 063511 (2000), J. J. Dalcanton, C. J. Hogan, *Astrophys. J.* **561**, 35 (2001).
- [55] G. B. Larsen, J. Madsen, *Phys. Rev. D* **52**, 4282 (1995). J. Madsen, *Phys. Rev. D* **64**, 027301 (2001);
- [56] D. Lynden-Bell, *Mon. Not. Roy. Astron. Soc.* **136**, 101 (1967),
- [57] S. Tremaine, M. Henon, D. Lynden-Bell, *Mon. Not. Roy. Astron. Soc.* **219**, 285 (1986).
- [58] R. F.G. Wyse and G. Gilmore, arXiv:0708.1492; G. Gilmore *et. al.* *Astrophys. J.* **663**, 948 (2007).
- [59] S. Peirani *et. al.*, *Mon. Not. R. Astron. Soc.* **367**, 1011 (2006); S. Peirani, J. A. de Freitas Pacheco, astro-ph/0701292;
- [60] E. Romano-Diaz *et.al.*, *Astrophys. J.* **637**, L93 (2006); E. Romano-Diaz *et.al.*, *Astrophys. J.* **657**, 56 (2007); Y. Hoffman *et.al.* *Astrophys. J.* **671**, 1108 (2007). A. Lapi, A. Cavaliere, *Astrophys. J.* **692**, 1, 174 (2009).
- [61] I. H. Gilbert, *Astrophys. J.* **144**, 233 (1966); *ibid*, **152**, 1043 (1968).
- [62] J R Bond, A S Szalay, *Astrophys. J.* **274**, 443 (1983).
- [63] D. Boyanovsky, H. J. de Vega, N. Sanchez, *Phys. Rev. D* **78**, 063546 (2008).
- [64] L. Gao, T. Theuns, *Science* **317**:1527 (2007).
- [65] S. Dodelson, L. M. Widrow, *Phys. Rev. Lett.* **72**, 17 (1994). X. Shi, G. M. Fuller, *Phys. Rev. Lett.* **82**, 2832 (1999); K. Abazajian, G. M. Fuller, M. Patel, *Phys. Rev. D* **64**, 023501 (2001); K. Abazajian, G. M. Fuller, *Phys. Rev. D* **66**, 023526, (2002); G. M. Fuller *et. al.*, *Phys.Rev. D* **68**, 103002 (2003); K. Abazajian, *Phys. Rev. D* **73**,063506 (2006); F. Munyaneza, P. L. Biermann, *Astron. and Astrophys.*, **458**, L9 (2006). A. Boyarsky, O. Ruchayskiy, D. Iakubovsky, *JCAP* **0903**:005 (2009), A. Kusenko, hep-ph/0703116; *Int. J. Mod. Phys. D* **16**:2325, (2008), T. Asaka, M. Shaposhnikov, A. Kusenko; *Phys.Lett. B* **638**, 401 (2006); P. L. Biermann, A. Kusenko, *Phys. Rev. Lett.* **96**, 091301 (2006);
- [66] D. Gorbunov, A. Khmelnitsky, V. Rubakov, *JHEP* **0812**:055 (2008); F D Steffen, *Eur. Phys. J. C* **59**, 557 (2009) and references therein.
- [67] S. Profumo, *Phys. Rev. D* **78**:023507 (2008) and references therein.

- [58] M. Lattanzi, J. W. F. Valle, Phys. Rev. Lett. 99:121301 (2007). F. Bazzocchi et al., JCAP 0808:013 (2008).
- [59] M. Viel *et.al.* Phys. Rev. Lett. 100:041304 (2007), Phys. Rev. **D71** 063534 (2005); A. Boyarsky et al. arXiv:0812.3256.
- [60] A. D. Dolgov and S. H. Hansen, Astropart. Phys. **16**, 339 (2002); K. Abazajian, S. M. Koushiappas, Phys. Rev. **D74** 023527 (2006); K. Abazajian, G. M. Fuller, W. H. Tucker, Astrop. J. **562**, 593 (2001); C. R. Watson et al., Phys. Rev. **D74**, 033009 (2006); A. Boyarsky et al. Mon. Not. Roy. Astron. Soc. **387**, 1361 (2008), Astron. Astrophys. 471:51, (2007), S. Riemer-Sorensen, et al. Phys. Rev. D76:043524, (2007), S. Riemer-Sorensen, S. H. Hansen and K. Pedersen, Astrophys. J. **644**, L33 (2006); M. Loewenstein, A. Kusenko, P. L. Biermann, Astrophys. J. 700, 426 (2009).
- [61] R. Bernabei et al. (DAMA/LIBRA coll.) Eur. Phys. J. C56:333 (2008).
- [62] R. Bernabei et al. Mod. Phys. Lett. A23:2125 (2008) and Int. J. Mod. Phys. A21, 1445 (2006); P. Gondolo, G. Raffelt, Phys. Rev. D 79, 107301 (2009); M. Pospelov, A. Ritz, M. B. Voloshin, Phys. Rev. D78:115012 (2008).
- [63] C.E. Aalseth et al. Phys. Rev. Lett. 101, 251301 (2008); M. Fairbairn, T. Schwetz, JCAP 0901:037 (2009); D. Hooper et al. Phys. Rev. D79, 015010 (2009); C. Savage et al. JCAP 04:010 (2009)
- [64] A. V. Tikhonov et al. Mon. Not. R. Astron. Soc. **399**, 1611 (2009).
- [65] H. Pagels, J R Primack, Phys. Rev. Lett. **48**, 223 (1982).
J R Bond, A S Szalay, M S Turner, Phys. Rev. Lett. **48**, 1636 (1982).
- [66] R. Cowsick and J. McClelland, Phys. Rev. Lett. **29**, 669 (1972).
- [67] B W Lee, S. Weinberg, Phys. Rev. Lett. **39**, 165 (1977). P Hut, Phys. Lett. **B 69**, 85 (1977). K. Sato, H. Kobayashi, Prog. Theor. Phys. **58**, 1775 (1977). M. I. Vysotsky, A. D. Dolgov, Ya. B. Zeldovich, JETP Lett. **26**, 188 (1977).
- [68] F. Munyaneza, P. L. Biermann, Astron. and Astrophys., 458, L9 (2006).
- [69] H. J. de Vega, N. Sánchez, arXiv:0907.0006. H. J. de Vega, P. Salucci, N. Sánchez, arXiv:1004.1908.
- [70] J. Sommer-Larsen, A. Dolgov, ApJ, 551, 608 (2001).
- [71] R. van de Weygaert, E. Platen, COSPA2008 review, arXiv:0912.2997.
- [72] Z. Ahmed et al., arXiv:0912.3592.
- [73] P.L. Biermann, et al., PRL 103:061101 (2009). P. Blasi, P. D. Serpico, PRL 103:051104 and 081103 (2009).